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RESEARCH ARTICLE

ANALYTICAL DETERMINATION OF STEADY STATE PERFORMANCE INDICES OF SINGLE-PHASE INDUCTION MOTOR, USING SYMMETRICAL COMPONENTS OF UNBALANCED VOLTAGES OF 3-PHASE SYSTEM

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ABSTRACT

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INTRODUCTION

The single-phase induction motor forms an integral part of all the various types of a.c motors and by far the most popular and has achieved a wide field of usefulness, especially on applications where the three-phase counterpart may not be economical or a poly-phase supply is unavailable[1]. The single-phase induction motor mainly consists of two major parts, viz, stator and rotor. The motor has two sets of winding that are identifiable as the field (stator) and armature (rotor) windings. As mentioned previously, the equivalent circuit of the motor is derived using the analogy of a three-phase induction motor whose one of its stator coils (windings) is disconnected.

Single-phase induction motor Concept

Obviously, if one line of a three-phase induction motor is opened by disconnection, while the motor is running with moderate load, the machine maintains running although at a slower speed. This condition is single-phase operation and it gives the implication that the three-phase induction motor has eventually become a single-phase induction motor [2].

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Department of Electrical of Engineering, NnamdiAzikiwe University Awka.Anambra State Nigeria The single-phase induction motor is analogous to polyphase induction motor counterpart in which a single-phase winding replaces the three-phase winding. A single-phase current in a single-phase winding produces a pulsating, not a rotating magnetic field. The pulsating magnetic fields produced by the single-phase induction motor can be resolved into two rotating magnetic fields with the help of double resolving magnetic field theory. All the magnetic fields have equal magnitude and they rotate in opposite directions.

The resultant torque is the sum of the two torques.

The equation of alternating magnetic field having axis fixed in space is represented by;

$$\propto (\tau) = \propto_{max} \sin \omega \cos \tau$$
 1

Where \propto_{max} = the amplitude of the sinusoidally distributed air gap influx.

Equation 1 can also be written as;

This work presents the steady-state performance indices of single phase induction motor, from

symmetrical components of unbalanced three-phase system. The analysis of unsymmetrical fault

conditions, using the method of symmetrical components, is a means of resolving an unbalanced

three-phase system of impedances into three equivalent single-phase systems with independence

impedance parameters. The implication of the above statement is that any unbalanced three-phase

system of voltages or currents can be regarded as due to the super-position of two symmetrical threephase systems having opposite phase sequence and a zero phase sequence, being equal to ordinary

single-phase current or voltage system. With the knowledge of the fore-going, an equivalent circuit for a single-phase induction motor is obtained when it is considered as a three-phase induction motor

with one of its stator windings disconnected. Values were assigned to the equivalent circuit

parameters. Torque/slip curves of a normal single phase induction motor with slip range $0 \le S \le 2$

$$\propto (\tau) = \frac{1}{2} \propto_{\max} \sin(\omega t + \tau) + \frac{1}{2} \propto_{\max} \sin(\omega t - \tau)$$

It can be observed that equation 2 has two components. The first component represents the revolving field moving in the negative direction while the second component represents the

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revolving field moving in the positive direction all having amplitude equal to $\frac{1}{2} \propto_{max}$.

The fields in the positive and negative direction are known as forward and backward rotating fields, and both rotate at synchronous speed ω s. The resulting instantaneous torque developed due to the revolving field has four components, viz;

- A torque due to the interaction of the forward travelling stator winding and rotor winding magneto-motive force (mmf) distributions.
- A torque due to the interaction of the backward travelling stator and rotor winding mmf distribution.
- A torque due to forward travelling stator winding mmf distribution and the backward travelling rotor winding mmf distribution.
- A torque due to the backward-travelling stator winding mmf distribution and the forward travelling rotor winding mmf distribution.
- The first component gives steady non-pulsating torque acting on the rotor in the forward direction and gives rise to a component torque/slip characteristic of the form obtainable from a polyphase induction motor [3]. The second component gives rise to a similar torque/slip characteristic, the torque acting in the opposite, backward direction. The third and fourth components give rise to torque which pulsate at twice supply frequency and do not contribute to the mean torque of the motor. That is oppositely travelling mmf distribution do not contribute to the mean torque.

Slip of Single Phase Induction Motor

The Slip of the rotor of Single-phase induction motor with respect to forward rotating field is given by;

$$s_{f} = \frac{\omega_{s} - \omega_{r}}{\omega_{s}} = 1 - \frac{\omega_{r}}{\omega_{s}} = S$$

$$3$$

Where, ω_s and ω_r are the synchronous speed and rotor speed respectively.

Similarly, when the rotor rotates in the opposite direction, the backward slips_bis given by;

$$s_b = \frac{\omega_s - (-\omega_r)}{\omega_s} = \frac{\omega_s + \omega_r}{\omega_s} = 1 + \frac{\omega_r}{\omega_s}$$

From equation, 3and 4, we can write;

$$s_b = 1 + (1 \quad S) = 2 \quad S \qquad 5$$

Equation 3 and Equation 5 represent the motor operation and the braking region.

Equivalent Circuit of Single-phase induction motor

This forms the bone of contention of this paper. The equivalent circuit of the motor is derived, using symmetrical components of unbalanced three-phase systems approach. The derivation is obtained on the assumption that the motor is a three-phase type with one of its stator windings disconnected as illustrated in Fig. 1 below:





From fig 1b, it can be inferred that;

$$\begin{vmatrix} I_R = I \\ I_Y = -I_R = -I \\ I_B = 0 \end{pmatrix}$$

$$V = V_{RS} + (-V_{YS}) (as V_{BS} = 0)$$
$$V_{RS} = -V_{YS} = -\frac{1}{2}V(asZ_{RS} = Z_{YS})$$
7

Where I = Input current, I_R = current at the Red phase, I_Y = Current at the Yellow phase, I_B = current at the blue phase, V_{RS} =voltage drop across Red phase of the stator winding, V_{YS} = Voltage drop across the yellow phase of the stator winding. In line with C.L Fortesque theorem, the symmetrical component of fig. 1a can be shown as the sum of fig. 2 (a, b, c) below:



Fig 2a. Positive Sequence Phasor



Fig. 2b. Negative Sequence Phasor



Fig. 2c. Zero Sequence Phasor

For the positive (+) sequence Phasor, taking the Red (R) Phasor as reference phasor as in fig. 2a, we obtain that;

I_{IR}= I_{IR}e^{jo}= I_{IR}(Reference phasor)

$$I_{1Y} = I_{1R} < 120 = a^2 I_{1R}$$
$$I_{1B} = I_{1R} < 120 = a I_{1R}$$

Similarly, for the negative (-) sequence quantities (Fig. 2b)

$$I_{2R} = I_{2R}e^{J^{0}} = I_{2R}(1+j0)$$

(Reference phasor)

$$I_{2Y} = I_{2R} < 120 = a I_{2R}$$

$$I_{2B} = I_{2R} < -120 = a^2 I_{2R}$$

Also, for the zero (0) phase sequence (Fig. 2c),

$$R_0 = Y_0 = B_0$$
 10

where a = phase sequence operator.

For the unbalanced system of current in Fig.1,

$$I_{R} = I_{1R} + I_{2R} + I_{0R}$$

$$I_{Y} = I_{1Y} + I_{2Y} = a^{2}I_{1R} + aI_{2R} + I_{0R}$$
11

The compact form of equation 11 can be put in matrix form as below:

$$\begin{bmatrix} I_{R} \\ I_{Y} \\ I_{B} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} 1_{OR} \\ 1_{IR} \\ 1_{2R} \end{bmatrix}$$
12

The inverse form of equation 12 is given as;

$$\begin{bmatrix} 1_{OR} \\ 1_{IR} \\ 1_{2R} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}$$
13

N.B: Equations (10-13) hold for voltages also

But from Fig. 1.

$$\mathbf{V} = \mathbf{V}_{\mathrm{RS}} + - \mathbf{V}_{\mathrm{YS}}$$

= V_{RS} - V_{YS} (as in equation 7).

In phase sequence representation;

$$V_{RS} = V_{R+} + V_{R-} + V_{RO}$$
 14

Similarly;

$$V_{YS} = V_{Y+} + V_{Y-} + V_{YO}$$
 15

16

But from equation 6;

$$V_{R+} = V_{IR} = \frac{1}{3} (V_{RS} + aV_{YS} + a^2 V_{BS})$$
$$V_{R-} = V_{2R} = \frac{1}{3} (V_{RS} + aV_{YS} + aV_{BS})$$

As terminal B is disconnected, $V_{BS} = 0$

Hence, equation 16 yields;

$$V_{R+} = \frac{1}{3} (V_{RS} + aV_{YS})$$

= $\frac{1}{3} (V_{RS} - aV_{RS}) = \frac{1}{3} V_{RS} (1-a)$ 17
 $V_{R-} = \frac{1}{3} (V_{RS} + a^2 V_{YS}) = \frac{1}{3} (V_{RS} - a^2 V_{RS})$

$$=\frac{1}{3} V_{\rm RS} (1-a^2)$$
 18

Also
$$V_{RO} = \frac{1}{3}(V_{RS} + V_{YS} + V_{BS})$$

= $\frac{1}{3}(V_{RS} + V_{YS}) = \frac{1}{3}(V_{RS} - V_{RS}) = 0$ 19

Substituting equations (17-19) into equation 14, we have;

$$V_{RS} = \left[\frac{1}{3}V_{RS}(1-a) + \frac{1}{3}V_{RS}(1-a^2) + 0\right]$$
$$= \frac{1}{3}V_{RS}(1-a)(a+2) = (a+2)V_{R+}$$
20

Similarly, from equation 15,

 $V_{Y+,} V_{Y-}$ and V_{YO} can be obtained as below;

For + sequence phasor (taking the yellow phasor as the reference phasor), we have

$$|_{1Y} = |_{1Y}e^{j0} = |_{1Y}(reference)$$
phasor)
$$|_{1B} = a^{2}|_{1Y} = |_{1Y}\angle -120^{0}$$

$$|_{1R} = a|_{1Y} = |_{1Y}\angle 120^{0}$$
21

For - sequence quantities,

$$|_{2Y} = |_{2Y}(1 + j_0) = |_{2Y}(reference phasor) |_{2B} = |_{2Y} \angle 120^0 = a|_{2Y} |_{2R} = |_{2Y} \angle -120^0 = a^2|_{2Y} |_{2R} = |_{2Y} \angle -120^0 = a^2|_{2Y} |_{2R} = |_{2Y} \angle -120^0 = a^2|_{2Y} |_{2R} = |_{2Y} |_{2R} = |_{2Y} \angle -120^0 = a^2|_{2Y} |_{2R} = |_{2Y} |_{2R} = |_{2Y} |_{2R} |_{2R} = |_{2Y} |_{2R} |_{$$

But,

$$I_{Y} = I_{1Y} + I_{2Y} + I_{0Y}$$

$$I_{B} = I_{1B} + I_{2B} + I_{0B} = a^{2}I_{1Y} + aI_{2Y} + I_{0Y}$$

$$I_{R} = I_{1R} + I_{2R} + I_{0R} = aI_{1Y} + a^{2}I_{2Y} + I_{0Y}$$
23

Equation 23 in its compact form yields

$$\begin{bmatrix} I_{Y} \\ I_{B} \\ I_{R} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} I_{0Y} \\ I_{1Y} \\ I_{2Y} \end{bmatrix}$$
 24

Inverting the matrix of equation 24 yields;

$$\begin{bmatrix} I_{0Y} \\ I_{1Y} \\ I_{2Y} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_Y \\ I_B \\ I_R \end{bmatrix}$$
25

Solving for V_{Y^+} , V_{Y^-} and V_{YO} as in V_{R^+} , V_{R^-} and V_{RO} yields;

$$V_{Y+} = \frac{1}{3} (V_{YS} + aV_{BS} + a^{2}V_{RS})$$

$$= \frac{1}{3} (V_{YS} + a^{2}V_{RS})$$

$$= \frac{1}{3} (a^{2}V_{RS} - V_{RS})$$

$$= \frac{1}{3} V_{RS} \quad (a^{2} - 1)$$

$$V_{Y-} = \frac{1}{3} (V_{YS} + a^{2}V_{BS} + aV_{RS})$$

$$= \frac{1}{3} (aV_{RS} - V_{RS})$$

$$= \frac{1}{3} V_{RS} (a - 1)$$

26

$$\therefore V_{YS} = V_{Y+} + V_{Y-} + V_{YO} = -\frac{(a+2)}{(1+a)} V_R$$
 27

The supply voltage V can be obtained by substituting equations 20 and 27 into equation 7 as below;

$$V = (a + 2)V_{R+-} \left[-\frac{(a+2)}{(1+a)}V_{R-} \right]$$
$$= (a + 2)V_{R+} + \frac{(a+2)}{(1+a)}V_{R-}$$
28

Taking the same steps as in the symmetrical components of voltages, the symmetrical components of the currents are given below;

$$I_{R+} = \frac{1}{3}(I - aI)$$

= $\frac{1}{3}I(I - a)$
$$I_{R-} = \frac{1}{3}(I - a^{2}I)$$

= $\frac{1}{3}I(I - a^{2})$
$$I_{R0} = 0$$

Transposing equation 29 for *I*, we have;

$$I_{+} = \frac{3I_{R+}}{1-a}$$

$$I_{-} = \frac{3I_{R-}}{(1-a^{2})}$$
30

29

The total input impedance of fig.1 is given by;

$$Z = \frac{V}{I}$$
 31

Substituting equations 28, and 30 into equation 31, yields;

$$\frac{VR+}{IR+} + \frac{VR-}{IR-}$$
$$Z = Z_{+} = Z_{-}$$
32

where Z_{+} = Positive phase sequence impedance

 Z_{-} = Negative phase sequence impedance

For a three-phase induction motor, the phase-sequence network with positive + and negative – sequence is shown in fig3 (a/b) below;



Fig 3-(a) + sequence of 3-phase induction motor at stand still. (b) – sequence of 3-phase induction motor at stand still

where R_s = Resistance of main (stator) winding X_s = Leakage reactance to main (stator) winding. X_m = Magnetizing reactance $R_{r'}$ = standstill rotor resistance to main (stator) winding. $X_{r'}$ = standstill rotor leakage reactance to main (stator) winding. I_{R+} , I_{R-} = main winding current. V = Applied voltage

To obtain the equivalent circuit of the single-phase induction motor, the positive (+) and negative (-) phase sequence networks of fig. 3, must be connected in series, noting that only V_{R+} produces the voltage source as in Fig. 4.



Fig. 4. Equivalent circuit of single-phase induction motor at standstill (S = 1)

Since the pulsating magnetic field is resolved into backward and forward fields having equal and opposite fluxes with the motor [2], the magnitude of each rotating flux is one-half of the alternating flux. Therefore, it is assumed that the two rotating fluxes are acting on two separate rotors. Hence, we then assume that the single-phase induction motor consists of two rotors having a common stator winding and two imaginary rotors as in fig. 5. At standstill, the impedance of each rotor referred to stator winding is



Fig. 5. Equivalent Circuit of Single phase induction motor for given slip S (running condition)

Performance indices of Single phase induction motor

The performance indices (characteristics) of a single-phase induction motor can be determined from its equivalent circuit of fig 5 for different values of slip ranging from $0 \le S \le 2$. The pertinent data of the machine are given in table 1 below.

Table 1. Parameters for single-phase induction motor

Parameter	Value
$\mathbf{x}_{\mathbf{s}} = \mathbf{x}_{\mathbf{r}'}$	1.82 Ω
x _m	35.82 Ω
$\mathbf{R}_{s} = \mathbf{R}_{r}$	1.10Ω
V	220 Ω
F	50H _z
No of poles	2

The e.m.f induced in the main (stator) winding by the forward and backward fluxes are E_f and E_b respectively.

The resultant induced e.m.f E_s in the Stator winding is given by;

$$E_s = E_f + E_h \tag{33}$$

Since the circuits of rotors due to the forward and backward fields are identical at stand still (S = 1), we can have that;

$$E_{f} = E_{b} \qquad 34$$

The rotor impedance Z_f due to forward field is;

$$Z_{f} = R_{f} + jx_{f} = \left(\frac{R_{r'}}{2s} + \frac{jX_{r'}}{2}\right) / / \left(\frac{jx_{m}}{2}\right)$$

$$35$$

The rotor impedance Z_b due to backward field is;

$$Z_{b} = R_{b} + jX_{b}$$
$$= \left(\frac{R_{r'}}{2(2-s)} + \frac{jX_{r'}}{2}\right) / / \left(\frac{jx_{m}}{2}\right)$$
36

Hence equation 33 yields;

$$E_{s} = I(Z_{f} + Z_{b})$$
37

But V = I $(Z_s + Z_f + Z_b)$ 38

where $Z_s = R_s + jx_s$

$$I_{rf} = \frac{E_{f}}{\left[\frac{R_{r'}}{2s} + \frac{jx_{r'}}{2}\right]} = \frac{IZ_{f}}{\sqrt{\left[\left[\frac{R_{r'}}{2s}\right]^{2} + \left[\frac{X_{r'}}{2}\right]^{2}\right]}}$$
39

$$I_{rb} = \frac{E_{b}}{\left[\frac{R_{r'}}{2(2-s)} + \frac{|X_{r'}|}{2}\right]} = \frac{IZ_{b}}{\sqrt{\left[\left[\frac{R_{r'}}{2(2-s)}\right]^{2} + \left[\frac{X_{r'}}{2}\right]^{2}\right]}}$$
40

Air gap power (Gross Power)due to forward field at the rotor winding;

$$P_{gf} = (I_{rf})^2 R_f = (I_{rf})^2 \left(\frac{R_{r'}}{2s}\right) \text{watts}$$

$$41$$

Air gap power (Gross power due to backward field at the rotor winding;

$$P_{gb} = (I_{rb})^2 R_b$$
$$= (I_{rb})^2 \left(\frac{R_{r'}}{2 (2-s)}\right) \text{ watts}$$

Mechanical Power output for the forward field;

$$P_{\text{mech f}} = [1 - (s)]P_{\text{gf}} = (1 - s)P_{\text{gf}}$$
$$= \frac{(1 - s)\frac{(l_{\text{rf}})^2 R_{\text{r}'}}{2s}}{(\text{watts})}$$
(43

Torque due to forward field

$$\tau_{f} = \frac{1}{\omega_{s}} p_{gf} = \frac{1}{\omega_{s}} \frac{(I_{rf})^{2} R_{r'}}{2s} (Nm)$$
44

A plot of the forward torque τ_f for slip range (0 \leq S \leq 2) is shown in fig 6a.

Similarly; mechanical power output for the backward field,

$$P_{mech,b} = [1 - (2 - s)]P_{gb}$$

= -(1 - s)P_{gb}
= -(1 - s)(I_{rb})^2 (\frac{K_r}{2(2-s)})_{watts} 45

Torque due to backward field

$$\tau_{\rm b} = \frac{-1}{\omega_{\rm s}} P_{\rm gb} = \frac{-1}{\omega_{\rm s}} \frac{(I_{\rm rb})^2 R_{\rm r'}}{2 (2-s)} ({\rm Nm})$$
46

NB (- Sign is due to opposite direction of the field to the forward field. Henceproducing a negative torque. A plot of the backward torque τ_b for slip range ($0 \le S \le 2$) is shown in fig 6b The sum of equation 43 and 45 is the net mechanical power output of the machine.

$$P_{\text{mech, net}=}(1-s)P_{\text{gf}+}[-(1-s)P_{\text{gb}}]$$

= $(1-s)[P_{\text{gf}} - P_{\text{gb}}]$
= $P_{\text{mech f}} + P_{\text{mech,b}}$ 47

Resultant Torque developed

$$\tau_r = \tau_f + \tau_b = \frac{P_{mech, net}}{\omega_s(1-s)} = \frac{P_{gf} - P_{gb}}{\omega_s}$$

Where $\omega_s = 2\pi n_s$,

$$n_{\rm s} = \frac{120 f}{p}$$

The motor efficiency is given by

$$\varepsilon\% = \frac{\text{output power}}{\text{input power}} \ge 100$$

= Pmech net x 100

$$=\frac{(1-s)(P_{gf}-P_{gb})}{VI\cos}x\ 100$$
48

Motor losses are assumed to be neglected.









Fig 6 (a – d). are plots of torque against slip for the induction motor a – Plot of torque/slip due to forward field b – Plot of torque/slip due to backward field c – Plot of torque/slip due to net field of a and b d – Plot of torque/slip when a, b, and c are superimposed



Fig 7.

It is evident from equation 38 that the motor current is given by;

$$I = \frac{V}{Z_s + Z_f + Z_h}$$
⁴⁹

The motor power factor

$$\cos = \frac{R_{s} + R_{f} + R_{b}}{Z_{s} + S_{f} + Z_{b}}$$
$$= \frac{R_{s} + R_{f} + R_{b}}{(R_{s} + j_{xs}) + (R_{f} + j_{xf}) + (R_{b} + j_{xb})}$$
50

A plot of the efficiency ε % against slips for the motor is shown in fig 7d. It is seen that the efficiency is low compared to a three-phase counterpart of equal rating.

Conclusion

The equivalent circuit of a single-phase induction motor from which the performance characteristic can be predicted has been presented. The torque-slip curve of the motor shows that at stand still (s = 1, $n_r = 0$), the net torque of the motor passes through zero, which depicts that the machine cannot start on its own, unlike the three-phase induction motor counterpart. More still, the torque developed by this type of motor drops to zero at a slip slightly below synchronous, due to negative torque developed by the backward field. Hence, the performance characteristics of single-phase induction motor are less satisfactory when compared with that of a three-phase induction motor counterpart of same rating [4]. The efficiency of single-phase induction motor is much lower as compared to three-phase motor, because of higher losses. It is seen from fig 7 that the maximum efficiency of the motor is low at about 57%. The power factor of any induction motor depends upon the magnitude of magnetizing current drawn by the motor. The magnetizing current in case of single-phase induction motor may be of the order of 50 to 60 percent of the full load current. As such, the power factor of single phase induction motor is much lower compared to three-phase motors.

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