RESEARCH ARTICLE

DIDACTIC STRATEGY INTHE TEACHING-LEARNING PROCESS OF DIFFERENTIAL CALCULATION WITH THE USE OF GEOGEBRA MATHEMATICAL ASSISTANT IN THE 12TH GRADE AT POLYTECHNICAL INSTITUTE OF ADMINISTRATION AND MANAGEMENT OF ZAIRE

[1]Pedro Binda Manuel and *Marcos João Púcuta[2]

[1]Master in Mathematics Teaching at Higher Institute of Education Sciences of Cabinda
[2]Assistant Professor at Mathematics Teaching Department of Higher Institute of Education Sciences of Cabinda

ABSTRACT

This article presents the results of a research carried out on the use of GeoGebra Mathematical Assistant in the teaching-learning process of differential calculus. This content deals with variation, movement, quantities that change and depend on other quantities, being considered by manypeopleasone of the great achievements of the human intellect. Differential calculus has a derivative concept considered fundamental for its study, as it is a linear operator, which forms a new function from the original, in which each point of the new function is the displacement of the original one. Its teaching has raised several difficulties for Secondary School students, such as: differential concept and derivative of a function at a point, geometric interpretation of derivative, rules of derivation and its applications in solving exercises and mathematical problems, lack of mathematical skills, determination of the growth and decrease intervals of a function, among others. These difficulties were identified through questionnairesand interviews carried out onteachers and 12th grade students, whose data were processed by the SPSS statistical software. The results showed that the visualization provided by GeoGebra in two-dimensional space proved to be an indispensable component in the construction of derivative concept. Based on these results, a didactic strategy was developed to favor the teaching-learning process of differential calculus using GeoGebra Mathematical Assistant in order to contribute to reducing these difficulties.

INTRODUCTION

In the current context, the teaching-learning process and human progress have shown new needs and challenges to pedagogical practice and its participation in the transformation of society. In this approach, Mathematics stands out as one of the sciences that relate everyday practices, with the nature of human reasoning and with numerical logic. Furthermore, it develops a constant search for the veracity of facts through precise and exact strategy and techniques in the areas of Medicine, Engineering, Physics, Chemistry, Economics, Astronomy, Architecture, among others. Rodrigues (2004, p.67) highlights the importance of student relating Mathematics with everyday life in its broadest way, as it develops reasoning, guarantees the way of thinking, enables the creation and improvement of ideas, something that is directly linked to the attitudes that the student will have in society. Its teaching-learning, developed with many limitations, which is out of context with real life.

However, this way of learning reduces the logic of the elaboration of its concepts, limiting itself only to the exposition of contents. In this way, it leads the student to a banking education challenged by the Brazilian pedagogue Paulo Freire cited by Púcuta and Bifica (2006), when stating that the student should not be considered as a small deposit and the contents should not be decontextualized. Mathematics offers multiple possibilities and contributions in a decisive and significant way in the development of the individual's personality, so that he is able to take on scientific and technological challenges, in the current dispute of the country's economic, political and social development, conceived as a set of results, methods, procedures, algorithms, among others. In this preamble to economic development, differential calculus stands out as one of the contents of this subject, which on the one hand helps to solve problems related to the various sciences mentioned above;it deals with various phenomena, such as the fall of a body, growth population, economic balance, heat spread, among others. According to the research carried out on 12th grade teachers and students, it was evident that this topic raises many difficulties on the part of students, such as: concept of differential and derivative of a function at a point, geometric interpretation of the derivative, rules of derivation and its applications in solving mathematical

Corresponding author: Marcos João Púcuta
[2]Assistant Professor at Mathematics Teaching Department of Higher Institute of Education Sciences of Cabinda.

exercises and problems, lack of mathematical skills, determining the growth and decay intervals of a function.These difficulties can be minimized if teachers use adequate methodologies and means capable to favoring the process of developing students' thoughts. LaraapudMarzola (1995, p. 65) argue thatthe pedagogical concern is to find "means capable to favoring the process of knowledge construction, as it is through which the student will reach the most advanced levels of conceptual development. In fact, for students to understand these contents in a dynamic, motivating and fun way, the computer and Mathematical Assistants are introduced: GeoGebra, Matlab, Maple, Descartes, Máxima, Graph, among others, as valuable resources that allow them to obtain, produce and combine their knowledge. GeoGebra is a Mathematical Assistant that facilitates the introduction and understanding of derivative concept and graphical sketch with greater precision. This research aims to contribute to the teaching-learning processimprovement of differential calculus using GeoGebra Mathematical Assistant in the 12[th]grade at Polytechnic Institute of Administration and Management of Zaire.

The teaching-learning process of differential calculus and its didactic foundations: Didactics is a science whose study object is the teaching-learning process, which takes place in a systematic and organized way at school. In addition, several authors have highlighted various interests in studying Didactics. Among them, the following stand out: Santos (2003), Libâneo (1992, p. 25), among others.Santos (2003) for instance, defines Didactics as an area responsible to investigate the foundations, conditions and ways of carrying out instruction and teaching, which takes on the character of "education science and takes the place ofPedagogy itself". According to Libâneo (1992, p. 25) Didactics is the main branch Pedagogy study which investigates the foundations, conditions and ways of carrying out instruction and teaching. There are many researchers who take Vygotsky's historical-cultural focus (1987, p. 34) as a premise for their developing conception, such as: Leóntiev (1978), Galperin (1975. p. 88), Talízina, (1987, p. 15-56), among others, where the role played by the Zone of Close Development (ZPD) within the teaching-learning process of Differential Calculus is recognized. In this process, the student is placed in the center of attention according to the results obtained from the diagnosis made throughout the research. In the teaching-learning process of Differential Calculus, the teacher-student-content relationship and the didactic elements (objectives, contents, methods, teaching means, (organization and evaluation form) must be taken into account.

Pimenta and Ghedin (2002) highlight the teacher's role in the classroom is to empower students with learning strategies that can be used from the beginning schooling and continues in higher education and later in professional life. Student is the individual who receives training and instruction from one or more teachers to acquire or expand their knowledge through an educational institution, that is, a school or any other place that offers conditions for the learning environment. The objective category is considered fundamental in the teaching-learning process of differential calculus, where the teacher must provide the student with a knowledge system that allows him to develop his mathematical skills to contribute to the development of society. Content is a set of knowledge from different sciences through which the teacher exposes their knowledge in the classroom.

In order to understand efficiently the differential calculus in the classroom, it is necessary the teacher to use the teaching methods. Then, Method is the way to reach the goal efficiently, in the teaching plan. It must be accompanied by the teaching means in order to facilitate students in the learning of differential calculus.The teaching means are all the materials used by the teacher to develop the class. Hence, for the correct use of these means it is necessary to organize them. Organizational forms are the different ways that the teacher uses to organize the teaching content. In order to verify the objectives, planned contents, methods, means and of organizationforms, it is necessary to associate them with the assessment. Assessment is a process to verify and assess the fulfillment of the proposed objectives and the didactic direction of teaching and learning, in its guidance and execution moments (ZILBERSTEIN, 2002). Differential Calculus is a linear operator, which forms a new function from the original, where each point of the new function is the displacement of the original one.This content is considered difficult by most 12th grade students at Polytechnic Institute of Administration and Management of Zaire in the following options: IT Management Technician, Business Management Technician, Accounting Technician, Public Administration Technician, Human ResourcesManagementTechnician, Library Technician, files and documentation, Commerce Technician and Finance Technician, because teachers do not use the best strategies and adequate methods that help students understand the tangent line, function, limit, differentialconcepts, derived from a point function, geometric interpretation of the derivative concept, derivation rules and their applications in solving exercises and problems in different areas, such as: Physics, Chemistry, Biology, Economics, Architecture, among others.

Villarreal (1999, p. 7), based on his experience with students from different levels of education, states that "the derivative concept is shown as a notion that presents frequent and persistent difficulties for students and its understanding is fundamental in theCalculation" classes. One of the possible causes of students' difficulties in learning this concept may be related to the difficulties in learning limits, which, as a consequence, lead to difficulties in derivatives, resulting from the fact that the derivative is a limit. The difficulty in understanding the concept of derivative of a function at a point when formally defined, based on the concept of limit, was identified by D'Avoglio (2002 p. 27).They can be reduced if the teaching of differential calculus is associated with computer environments. Gravina and Santarosa (1998) state that computerized environments favor the learning process. They allow students to manipulate concrete objects and, consequently, enable the transformation into abstract objects so that, later, mental actions can be applied to them. Wilges (2006, p.25) emphasizes that "computerized environments arise for the educator and student to explore an alternative space for exchanging knowledge and for the knowledge"construction.

The Use of GeoGebra Mathematical Assistant in the teaching-learning process of Differential Calculus: GeoGebra Mathematical Assistant is a dynamic geometry program that illustrates the properties of geometric figures, reflection concept, translation and rotation (congruence) and homothety (similarity), calculate angles, derivative of one-variable functions, differential of one-variable functions, partial derivatives, plot points, lines, line segments, find the

slope of the line, areas between two functions using definite integrals, graphical sketch of trigonometric functions, exponential, logarithmic, polynomial, rational, irrational and modular, graphic representation of conic sections (circumference, parable, ellipse and hyperbola) and quadratic surfaces (ellipsoid, sphere, cylinder, one- and two-leaf hyperboloid, parable), among other mathematics and statisticscontents. This Mathematical Assistant streamlines the Geometry and Algebrastudy, in order to facilitate the investigation and learning of their mathematical concepts, theorems and demonstrations. These features allow the application to be used as a pedagogical resource in Primary and Secondary education. Púcuta (2019) states that GeoGebra is one of the most used software in the teaching-learning process of Mathematics in all continents of the world.

This Mathematical Assistant brings benefits to students and teachers, especially in the visualization of the derivative of a functionconcept, among others, which is one of the aspects. Visualization allows the achievement of goals as important as the students' motivation in learning certain mathematical content. Arcavi 2003,p.217 considers visualization as the skill, process and product of creation, interpretation, use of reflection on figures, images, diagrams, in our minds, on papers or with technological tools, for the purpose of describing and communicating information, thinking about and developing previously unknownideas and advanced understanding.Mathematical Assistant Geogebracontains three zones. The first is called the input zone or command. The Mathematical functions, symbols, and expressions are introduced in this zone. The second is known as the algebraic zone: in this zone the mathematical functions and expressions entered in this command bar are presented in free and independent objects. Finally, the third is called the graphics zone. It shows the graphics of the functions entered in the input bar.

The figure below shows the referenced zones.
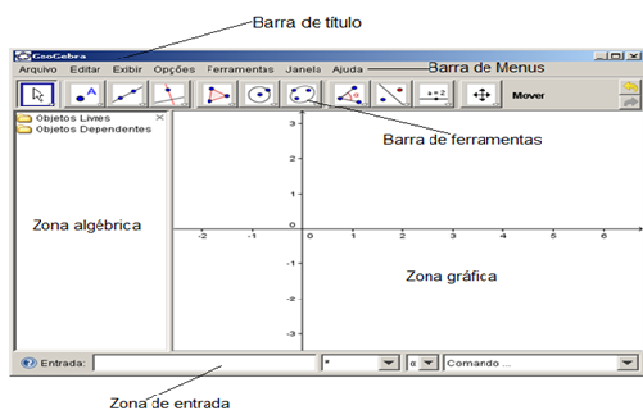
As it can be seen in the above figure, each icon on the title bar has several options, related to the functions described in its drawing. These options are activated by clicking the arrow in the lower right corner of each icon. To reactivate the algebraic part, just go to the "View" item in the menu and click on "Algebra Window". In that same item, you can activate/deactivate the axes, the mesh and the construction protocol. The following is an illustrative example of the use of GeoGebra Mathematical Assistant in the introduction of derivative functionconcept$f(x) = senx$ made by 12[th]grade students at Polytechnic Institute of Administration and Management of Zaire when evaluating their knowledge acquired in Mathematics classes on the Differential calculus

topic.When using this computer program, they followed certain steps:

1st Step: Open the program.
2nd Step: Insert the function $f(x) = sen(x)$in the input zone and press Enter.
3[rd]Step: Select the New Point tool and click on the graph of the function $f(x)$ to create point A on the graph of $f(x)$.
4[th]Step: Activate the tangent tool and click on the point A and on the graph of $f(x)$rename T the tangent (right click on it and rename it).
5th Step: Select the Elige and Move tool and drag point A.
6th Step: Activate the pendant tool and click on the tangent drawn at point A.
7th Step: Select the Elige and Move tool and drag point □, then observe the movement of the tangent and the corresponding slope m.
8th Step: Insert $B = (x(A), m)$and activate the trace of this point (right click on B).
9th Step: Select the Elige and Move tool and drag the point □, then observe the movement of the tangent and the corresponding slope m.
10th Step: Insert the derived commands$[f]$in the input bar and click in order to have the derivative of that function which is $f'(x) = cos(x)$
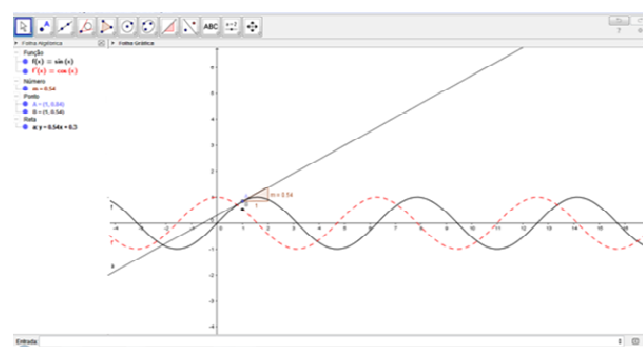
The following is the figure resulted from the above steps.

Figure 2. Obtaining the derivative concept from the function
$$f(x) = senx$$

Didactic strategy in the teaching-learning process of Differential Calculus: Strategy is a topic addressed by several researchers such as: Addine (2011), Vale Lima (2010), Quintana (2011), Stlano and Torres (2013), Púcuta (2016), among others. According to Vale Lima (2010), strategy is a set of sequential and interrelated actions that, starting from an initial state (given by the diagnosis), arrive at the ideal state as a result of planning. The strategy has the following components: the mission, objectives, actions, methods and procedures, resources, people responsible for the actions and time in which they must be carried out, theimplementation and evaluation forms. Each component plays a role. In the mission the most general social ends are expressed. Objectives express what must be achieved in the development of the work in a given period of time. The mission and objectives are elaborated according to the results of the diagnosis. In the stages, the objective and specific actions are elaborated. Actions are tasks that must be performed in order to fulfill the outlined objectives. The implementation forms are actions that make it possible to put the strategy into practice; the evaluationform analyzes the strategy to issue opinions on its application and result process.
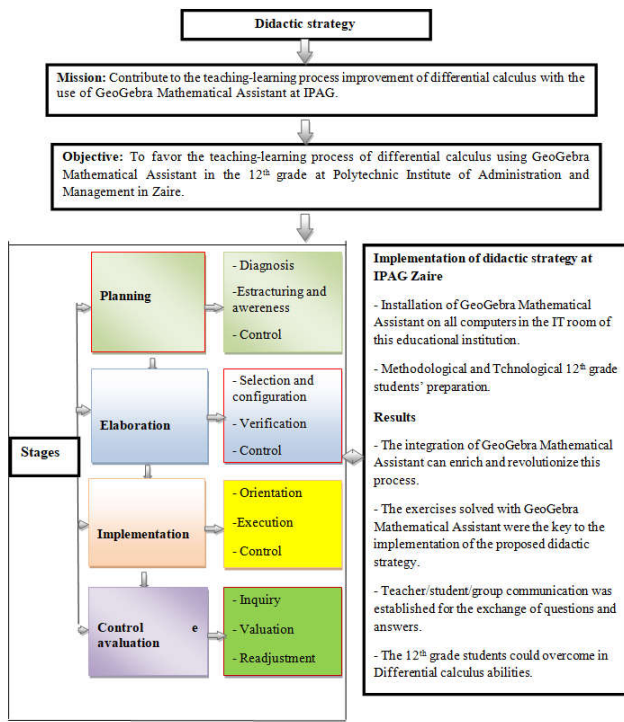
Source: Author's elaboration

Diagram 1. Diagram ondidactic strategy in teaching-learning process of Differential Calculus

METHODOLOGICAL RESULTS

Population, Sample and Data Processing: As a result of this research, 164 elements were surveyed, among them 4 teachers and 160 12th grade students. A Questionnaire containing structured questions onthe teaching-learning of Differential Calculus was applied followed by interview where the results obtained were processed by the SPSS statistical software.

The implementation of the didactic strategy results: For its implementation in the 12th grade, it was necessary to create technological conditions and pedagogically prepare the students. The results obtained showed that the use of the GeoGebra Mathematical Assistant in the teaching-learning process of Differential Calculus considerably transformed the classroom environment, making students more dynamic, motivated, curious, participatory, attractive and minimized their difficulties in solving exercises and problems that lead to this content, as it is illustrated in the chart below referring to the comparative results of the Pre-Test and Post-Test applied to the participants at Polytechnic Institute of Administration and Management of Zaire throughout the research . The degree of reliability of the results obtained in the application of the questionnaire to Mathematics teachers is 0,850and that of the students is 0,775. These data demonstrate that there is internal consistency of the same and the instrument used to collect information concerning the teaching-learning of Differential Calculus is valid. The data in Graph 1 show that the results obtained by the students in the post-test surpassed those obtained in the pre-test, where 6% obtained Good, 31% Very Good and 63% of students obtained Excellent results. Based on this assumption, it can be said that the GeoGebra Mathematical Assistant is a more innovative educational resource to contribute to the full students integration, allowing the development of autonomous work, highlighting the role of graphic language, relativizing the importance of calculation and symbolic manipulation, contemplates different learning rhythms; it enhances the development of students'
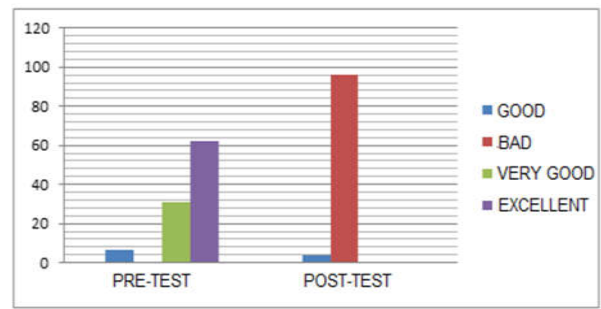
Graph 1. Comparative results of tests applied to 12th grade students

self-expression skills and broadens the horizons of mathematical content and gives them the pleasure of learning Differential Calculus.

Didactic strategy Validation by the experts: Doctoral Professors in Mathematics Teaching, who teach Mathematical Analysis in higher Education institutions, were consulted for its validation. They evaluated it according to the level of reliability of the questionnaire addressed to the teachers, which is quite good, having guaranteed the consistency of the data, the satisfactory results obtained by the 12th grade students in the post-test concerning the use of GeoGebra Mathematical Assistant in the introductionof the derivative function concept $f(x) = senx$,indicators related to the design and implementation of the didactic strategy, methodology used for the integration of GeoGebra Mathematical Assistant in the teaching-learning of Differential Calculus, its relevance and feasibility. Due to these aspects, the experts validated the didactic strategy as Excellent.

DISCUSSION

The implementation of the GeoGebra Mathematical Assistant in 12th grade at Polytechnic Institute of Administration and Management of Zaire has brought satisfactory results, which ensure its effectiveness in the teaching-learning process of Mathematics. However, GeoGebra is a powerful tool that expands students' knowledge regarding the study of straight lines, segments, conic sections, quadratic surfaces, calculation of areas using defined integrals, among other contents.

In this regard, Cavalcante (2015) states that GeoGebra has all the traditional tools of a dynamic geometry software: points, segments, straight lines and conic sections; equations and coordinates can be entered directly, showing the didactic advantage of two different representations of the same interactive object: the geometric and the algebraic representation. GeoGebra privileges the visualization of the concepts of limit and derivative and contributes to the realization of numerical experimentation and dynamism, emphasizing the interpretation of geometric constructions, which are difficult to be worked on in the classroom. Goldenberg (1998, p. 37) defends the idea that there are two types of visualization that students need to know, both in mathematical and other contexts. These concern the ability to create, manipulate and "read" mental images of common aspects of reality; visualize spatial and quantitative information and visually interpret the information presented to you; reviewing and analyzing steps previously taken as objects that could be touched and drawn and interpreted or made to appear.

The table that contains the steps, phases and actions developed in this didactic strategy is presented below

Nº	STAGES	PHASES	ACTIONS
1	**Planning**	Diagnosis	- Questions were asked about the technology availability on the use of the Geogebra MathematicalAssistant in the teaching-learning process of Differential Calculus. - Questions were asked about the teachers' preparation in the "Differential Calculus" content, as well as their computer skills and those of the students.
		Estruturation awereness	-The 12th grade Mathematics textbook and program were analyzed. - The Mathematics program was structured in accordance with its objectives.
		Control	- The results obtained from the pedagogical and technological diagnosis were processed.
2	**Elaboration**	Selection and configuration	- The methodological guidelines, exercise system and problems that lead to the Differential Calculus were introduced. - A set of practical activities related to Differential Calculus was elaborated.
		Verification	- It was verified if the planned activities, the system of exercises elaborated for the students are well defined and their objectives. - It was verified whether the use of GeoGebra Mathematical Assistant responds to the conceived model.
		Control	- The levels reached by the students in carrying out the actions elaborated on the Differential Calculus were valued.
3	**Implementation**	Guidance	- Each teacher, based on their mission and functions, guides the indications to be followed and provides guidance for fulfilling the Differential Calculus objectives.
		Execution	- The main GeoGebra Mathematical Assistant window and its commands were explored. - Differential and derivative functions were calculated using the GeoGebra Mathematical Assistant.
		Control	- The fulfillment level of the proposed objectives were verified.
4 4	**Control and assessment**	Inquiry	- The results obtained by the students using the GeoGebra Mathematical Assistant were integrated.
		Valuation	- The results obtained at each stage were valued, as well as the transformation level achieved by students and teachers using the GeoGebra Mathematical Assistant.
		Readjustment	- Adaptations and changes were made in the actions, through the positive aspects and students' difficulties detected throughout the research.

Source: Author's elaboration

CONCLUSION

GeoGebra mathematical software is an important pedagogical tool in the teaching-learning process development of differential calculus. Its use transformed the classroom environment, making students more dynamic, motivated, curious, participatory, and attractive and minimized their difficulties in solving exercises and problems that lead to this content, and favored the visualization of the geometric interpretation of derivative concept which is difficult to lead with in the classroom. The implementation of the didactic strategy in the 12th grade generated satisfactory results that show the advantage and importance of integrating GeoGebra Mathematical Assistant in the teaching-learning of Differential Calculus. In the validation of this strategy, several requirements were taken into account: the reliability level of the questionnaire addressed to Mathematics teachers, satisfactory results obtained by the students in the post-test, design and implementation of the didactic strategy, methodology used for the integration of the GeoGebra Mathematical Assistant in the teaching-learning of Differential Calculus, its pertinence and feasibility.

REFERÊNCES

Addine, F. y otros. *Didáctica General y su enseñanza en la Educación Superior Pedagógica. Aportes e impacto,* La Habana, Cuba. 2011

ARCAVI, A. The role of visual representations in the learning of mathematics. Education Studies in Mathematics, 2003. p. 217

Cavalcante, F. C. *Estudo qualitativo sobre tentativas e ideações suicidas com 60 pessoas.* Vol. 20, Rio de Janeiro. 2015

D`Avoglio, A. R. *Derivada de uma função num ponto: Uma forma significativa de introduzir o conceito. 2002.* p. 27

Freire, P. *Pedagogia da autonomia: saberes necessários à pratica educativa.* São Paulo, Brasil. 1996

Goldenberg, M. *Técnicas e métodos qualitativos.* Editora Record. 1998, p. 37

Galperim, A. P. *En que medida es aplicable el concepto caja negra en la psicologia del aprendizaje.* Editorial Puebloy educación. Habana.1975. Pág.88

Gravina, M. A. e Santarosa, L. M. C. *Formação de professores em informática na educação, actas do II congresso ibero-americano de informática na educação, Lisboa/Portugal.1998, Pág.8- 22-23.*

Lara, I. C. M. "*Jogando com a matemática* de 5° a 8° serie". – 1.ed – São Paulo: Rêspel, 2003.

Marzola, N. R. *A "reinventação da escola" segundo o construtivismo pedagógico: para uma problematização da mudança educacional.* Porto Alegre: UFRGS, 1995. Teste (Doutorado) – Programa de Pós-Graduação em Educação, Faculdade de Educação, Universidade Federal do Rio Grande do Sul, 1995 , p.65

Leóntiev, A. N. O d*esenvolvimento do psiquismo.* Lisboa,Livros Horizonte-Brasil. 1978

Valle Lima, A. D. *La investigación pedagógica. Otra mirada.* La Habana, Cuba, 2010, p. 188-195Paulo Freire

Pimenta, S. G.; Ghedin, E. *Professor reflexivo no Brasil: génese e crítica de um conceito.* São Paulo: Cortez, 2002.

Púcuta, M. J. e Bifica, N. *Tratamento metodológico de Tanto porcento numa perspectiva construtivista na 6ª classe. Trabalho de Fim de Curso apresentado para a obtenção do título de Licenciado em Ciências de Educação. Cabinda. 2006, Pág.15.*

Púcuta, M. J. *El proceso de la enseñanza-aprendizaje del Cálculo Integral y sus aplicaciones. Una estrategia didáctica con integración de las Tecnologías de*

Información y las Comunicaciones. Tesis doctoral en Ciencias Pedagógicas. Habana, Cuba. 2016.

Púcuta, M. J. *Material de Tecnologias de Informação e Comunicação para o Mestrado em Ensino da Matemática.* Cabinda, 2019

Quintana, A. V. *Estrategia didáctica para el proceso de enseñanza-aprendizaje del procesamiento de datos en la asignatura Matemática en la Educación Secundaria Básica.* Tesis presentada en opción al grado científico de Doctor en Ciencias Pedagógicas.La Habana, Cuba. 2011

Rodrigues, J. B. *Una propuesta metodológica para la utilización de las TIC en el proceso de enseñanza - aprendizaje de las funciones matemáticas. Tesis en opción al grado científico de Doctor en Ciencias Pedagógicas. Cuba.2004, Pág. 67.*

Santos, J. C. *Processos Participativos na Construção do Conhecimento em Sala de Aula.* Cáceres, MT: UNEMAT Editora, 2003.

Libânio, J. C. *A didáctica geral. Brasil. 1992, Pág. 25.*

Stlano, F. e Torres, P. *Estrategia didáctica para la enseñanza-aprendizaje de la asignatura Análisis de Sistemas mediado por las TIC para el desarrollo de las habilidades informáticas en la carrera de ingeniería en informática en Angola.* 2013

Talizina, N. *Exigencias a la teoría psicológica de la enseñanza como base de la dirección.* Moscou: Editorial Progreso. 1987, Pág. 15-56.

Villarreal, A. M. *Tecnologias digitais e educação Matemática. 1999, Pág.7.*

Vygotsky, L. *A estruturação de conhecimento Matemático.* Experiência cubana, 1987. Pág. 34.

Wilges, I. J. Finanças públicas: orçamento e direito financeiro. Editora AGE Ltda, 2006. , p.25

Zilberstein, J. y Sivestre, M. *Cómo hacr más eficiente el aprendizaje?.* México. Ceito. 2002. Pág. 107

****