



ISSN : 2350-0743



RESEARCH ARTICLE

CROSSREF

OPEN ACCESS

PERMANENCE OF A 3-SPECIES LOTKA-VOLTERRA MODELS WITH CAUTIOUS EFFECT AND FEEDBACK CONTROL

\*Qi Shang, Changyou Wang

College of Applied Mathematics, Chengdu University of Information Technology, Chengdu, Sichuan, 610225, P. R. China

ARTICLE INFO

Article History

Received 24<sup>th</sup> May, 2025  
Received in revised form 27<sup>th</sup> June, 2025  
Accepted 20<sup>th</sup> July, 2025  
Published online 30<sup>th</sup> August, 2025

Keywords:

Nonautonomous; Cautious effect;  
Feedback control; Predator-prey model;  
Persistence.

\*Corresponding author:  
Changyou Wang

ABSTRACT

In this study, we analyze a three-species Lotka-Volterra model that incorporates two critical extensions: cautious behavioral effects and feedback control mechanisms. The cautiousness parameter quantifies adaptive strategies adopted by species in response to environmental uncertainties and interspecific interactions, while feedback control variables represent human interventions or ecosystem management efforts. These modifications enhance the biological realism of the classical model by accounting for both dynamic behavioral adjustments and external regulatory influences. Through rigorous mathematical analysis, we employ the comparison principle for differential equations alongside newly developed analytical techniques to derive sufficient conditions for system permanence. This property, which ensures that all species maintain positive populations over time, serves as a fundamental indicator of ecological stability. Our results demonstrate that the interplay between cautious behavior and feedback control significantly influences long-term system dynamics. Specifically, we identify parameter thresholds where behavioral adaptations and management interventions collectively promote sustainable coexistence. The analytical framework developed in this study provides a systematic approach to examining how adaptive species behaviors and external controls jointly determine ecosystem persistence. These findings contribute to both theoretical ecology and applied conservation by offering quantitative guidelines for designing management strategies that account for complex interactions between biological adaptability and human influence.

Copyright©2025, Qi Shang, Changyou Wang. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Citation: Qi Shang, Changyou Wang. 2025. "Permanence of a 3-species Lotka-Volterra models with cautious effect and feedback control", International Journal of Recent Advances in Multidisciplinary Research, 11, (08), 11631-11638.

INTRODUCTION

The classical Lotka-Volterra models, originally formulated by Lotka [1] and Volterra [2], remain pivotal in theoretical ecology for studying predator-prey dynamics. Central to these models is the functional response concept, which quantifies the relationship between prey density and predation intensity. Traditional formulations such as Holling's types I-III [3] assume predation rates depend exclusively on prey availability. However, empirical evidence reveals that predator behavior is influenced by both prey abundance and conspecific density. This limitation was first addressed by Arditi and Ginzburg [4] in 1989 through predator-dependent functional response models, marking a paradigm shift in ecological modeling. Subsequent theoretical advancements have further refined these frameworks. Abrams [5] in 1990 demonstrated the significance of predator-prey biomass ratios in determining system stability. Using Jacobian matrix analysis, he showed that predation efficiency often declines under high predator densities due to intraspecific interference—a phenomenon now recognized as the ratio-dependent effect. Building on this, Skalski and Gilliam [6] in 2001 introduced predator interference coefficients into the Holling type II model, creating a more biologically realistic representation of density-mediated predation. Despite these improvements, real-world ecosystems exhibit additional complexities. Factors such as resource variability, environmental stochasticity, and parameter uncertainties (including mortality and survival rates) significantly impact population dynamics. A critical question in population ecology concerns ecosystem resilience: under what conditions can systems maintain stability when subjected to persistent disturbances? This challenge has motivated the integration of feedback control mechanisms into dynamical models, representing either natural regulatory processes or human management interventions. Recent studies have made significant progress in this direction. Liao et al. [7] in 2008 derived persistence criteria for discrete-time n-species nonautonomous Lotka-Volterra competition systems with feedback controls. Wang et al. [8] in 2018 established sufficient conditions for three-species competitive system persistence under similar control structures. These works provide important theoretical foundations for our current study. Behavioral ecology

research provides additional insights. Studies on rodent foraging behavior conducted by Brown et al. [9] reveal that species adjust predation strategies in response to environmental conditions and morphological constraints. Specifically, predators demonstrate cautious behavior by modifying predation rates, particularly under conditions of low prey density, as a mechanism to prevent resource depletion. This risk-benefit assessment in foraging decisions suggests that both predators and prey behave as rational agents capable of strategic adaptation: predators weigh hunting risks against nutritional gains, while prey employ evasive tactics when threatened. Given the limited research on Lotka-Volterra models incorporating prudent behavioral effects, this study develops a novel framework by integrating two key extensions: (1) cautious behavioral adaptations by both predators and prey, where individuals dynamically adjust strategies based on real-time population conditions; and (2) feedback control mechanisms representing external regulatory influences. By examining the interplay between these factors, we aim to establish sufficient conditions of system persistence for the following 3- species Lotka-Volterra models with caution effect and feedback control

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_1(t)x_1(t) - b_1(t)x_2(t) - \frac{k_1(t)x_1(t)x_3(t)}{p_1(t) + x_1(t)} - d_1(t)u_1(t)], \\ \dot{x}_2(t) = x_2(t)[r_2(t) - a_2(t)x_2(t) - b_2(t)x_1(t) - \frac{k_2(t)x_2(t)x_3(t)}{p_2(t) + x_2(t)} - d_2(t)u_2(t)], \\ \dot{x}_3(t) = x_3(t)[-r_3(t) - a_3(t)x_3(t) + \frac{\mu_1(t)k_1(t)x_1^2(t)}{p_1(t) + x_1(t)} + \frac{\mu_2(t)k_2(t)x_2^2(t)}{p_2(t) + x_2(t)} + d_3(t)u_3(t)], \\ \dot{u}_1(t) = e_1(t) - f_1(t)u_1(t) + q_1(t)x_1(t), \\ \dot{u}_2(t) = e_2(t) - f_2(t)u_2(t) + q_2(t)x_2(t), \\ \dot{u}_3(t) = e_3(t) - f_3(t)u_3(t) - q_3(t)x_3(t). \end{cases} \quad (1)$$

with the following initial values

$$x_i(t_0) = x_{i0} > 0, \quad u_i(t_0) = u_{i0} > 0, \quad i = 1, 2, 3. \quad (2)$$

The system (1) describes the interactions between two prey and one predator populations with caution effect and feedback control. Where  $x_1(t), x_2(t)$  and  $x_3(t)$  represent the population densities of two competing prey and one predator;  $r_i(t) (i = 1, 2)$  denote the intrinsic growth rate of the prey species at patch 1 and patch 2;  $r_3(t)$  is the death rate of the predator;  $a_i(t) (i = 1, 2, 3)$  are the population self-interaction;  $b_1(t)$  and  $b_2(t)$  are the interaction between patch 1 and patch 2;  $k_i(t) (i = 1, 2)$  are the capture rate of the predator;  $p_i(t) (i = 1, 2)$  express the caution effect of the predator when it feeds on the prey;  $\mu_i(t) (i = 1, 2)$  are the predator switching rate;  $u_i(t) (i = 1, 2, 3)$  are an indirect control variable;  $d_i(t), e_i(t), f_i(t)$  and  $q_i(t) (i = 1, 2, 3)$  are control parameters. The coefficients  $a_i(t), b_i(t), d_i(t), e_i(t), f_i(t), k_i(t), p_i(t), q_i(t), r_i(t), \mu_i(t)$  are positive continuous bounded functions of  $t$  for  $i = 1, 2, 3$ .

**Main results and proof:** Considering the biological plausibility of the system, we give the initial conditions  $x_i(t_0) > 0, u_i(t_0) > 0$ . Firstly, we introduce the following notation and definitions. Given a continuous and bounded function  $g(t)$  defined on  $[t, +\infty)$ , we set

$$g^m = \sup \{g(t) \mid t_0 < t < +\infty\}, \quad g^l = \inf \{g(t) \mid t_0 < t < +\infty\}.$$

**Definition 1.** System (1) is called permanent, if there exist positive constants  $M_i, N_i, m_i, n_i, (i = 1, 2, 3)$  and  $T$ , such that  $m_i \leq x_i(t) \leq M_i, n_i \leq x_i(t) \leq N_i$  as  $t > T$  for any positive solution  $(x_1(t), x_2(t), x_3(t), u_1(t), u_2(t), u_3(t))$  of the system (1) with the positive initial values.

For the system (1), we let

$$\begin{aligned} M_1^* &= \frac{r_1^m}{a_1^l}, M_2^* = \frac{r_2^m}{a_2^l}, N_3^* = \frac{e_3^m}{f_3^l}, \\ N_1^* &= \frac{e_1^m + q_1^m M_1}{f_1^l}, N_2^* = \frac{e_2^m + q_2^m M_2}{f_2^l}, \end{aligned}$$

$$M_3^* = \frac{\mu_1^m k_1^m M_1^2 p_2^l + \mu_2^m k_2^m M_2^2 p_1^l + N_3 d_3^m p_1^l p_2^l - r_3^l p_1^l p_2^l}{a_3^l p_1^l p_2^l},$$

$$m_1^* = \frac{p_1^l (r_1^l - d_1^m N_1 - b_1^m M_2)}{a_1^m p_1^l + k_1^m M_3}, m_2^* = \frac{p_2^l (r_2^l - b_2^m M_1 - d_2^m N_2)}{a_2^m p_2^l + k_2^m M_3},$$

$$n_1^* = \frac{e_1^l + q_1^l m_1}{f_1^m}, n_2^* = \frac{e_2^l + q_2^l m_2}{f_2^m}, n_3^* = \frac{e_3^l - q_3^m M_3}{f_3^m},$$

$$m_3^* = \frac{\mu_1^l k_1^l p_2^m m_1^2 + \mu_1^l k_1^l m_1^2 m_2 + \mu_2^l k_2^l p_1^m m_2^2 + \mu_2^l k_2^l m_1 m_2^2 - r_3^m p_1^m p_2^m - r_3^m p_1^m m_2 - r_3^m p_2^m m_1 - r_3^m m_1 m_2}{a_3^m (p_1^m + m_1)(p_2^m + m_2)},$$

where  $M_i, N_i, m_i, n_i$ , are some appropriate positive constants such that

$$0 < m_i < m_i^* < M_i^* < M_i, 0 < n_i < n_i^* < N_i^* < N_i, i = 1, 2, 3.$$

**Theorem 1.** Assume that the system (1) satisfies the following conditions

$$(H_1) \mu_1^m k_1^m M_1^2 p_2^l + \mu_2^m k_2^m M_2^2 p_1^l + N_3 d_3^m p_1^l p_2^l > r_3^l p_1^l p_2^l;$$

$$(H_2) r_1^l > d_1^m N_1 + b_1^m M_2; (H_3) r_2^l > d_2^m N_2 + b_2^m M_1; (H_4) e_3^l > q_3^m M_3;$$

$$(H_5) \mu_1^l k_1^l p_2^m m_1^2 + \mu_1^l k_1^l m_1^2 m_2 + \mu_2^l k_2^l p_1^m m_2^2 + \mu_2^l k_2^l m_1 m_2^2 > r_3^m p_1^m p_2^m + r_3^m p_1^m m_2 + r_3^m p_2^m m_1 + r_3^m m_1 m_2.$$

Then the system (1) is permanent.

**Proof.** According to the first and the second equations of system (1), we have

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)[r_1(t) - a_1(t)x_1(t) - b_1(t)x_2(t) - \frac{k_1(t)x_1(t)x_3(t)}{p_1(t) + x_1(t)} - d_1(t)u_1(t)] \\ &\leq x_1(t)[r_1(t) - a_1(t)x_1(t)] \\ &\leq x_1(t)[r_1^m - a_1^l x_1(t)] \\ &= a_1^l x(t) \left[ \frac{r_1^m}{a_1^l} - x_1(t) \right]. \end{aligned}$$

In view of the comparison theorem, one has

(A) When  $0 < x_{10} < M_1^*$ , it holds that  $x_1(t) < M_1^* < M_1$ , for  $t \geq t_0$ .

(B) When  $x_{10} \geq M_1^*$ , it hold that  $x_1(t) < M_1$  for a sufficiently large  $t$ . Otherwise, if  $x_1(t) > M_1$ , then there exist  $\alpha > 0$  such that  $x_1(t) \geq M_1^* + \alpha$ . Moreover, we have

$$\dot{x}_1(t) \Big|_{x_1(t) > M_1} \leq x_1(t)[r_1(t) - a_1(t)x_1(t)] \leq a_1^l x_1(t)[M_1^* - x_1(t)] < -\alpha a_1^l x_1(t),$$

Integrating the above inequality yields

$$x_1(t) < x_{10} \exp(-\alpha a_1^l (t - t_0)).$$

Thus, it holds that  $x_1(t) < x_1(t_0) \exp(-\alpha a_1^l (t - t_0)) \rightarrow 0$  as  $t \rightarrow +\infty$ . This is a contradiction, so there exist a sufficiently large  $T_1 \geq t_0 \geq 0$  such that

$$x_1(t) < M_1 \text{ as } t > T_1. \tag{3}$$

Similarly, from the second equation of system (1), one has

$$\begin{aligned} \dot{x}_2(t) &= x_2(t)[r_2(t) - a_2(t)x_2(t) - b_2(t)x_1(t) - \frac{k_2(t)x_2(t)x_3(t)}{p_2(t) + x_2(t)} - d_2(t)u_2(t)] \\ &\leq x_2(t)[r_2(t) - a_2(t)x_2(t)] \\ &\leq a_2^l x(t) \left[ \frac{r_2^m}{a_2^l} - x_2(t) \right], \end{aligned}$$

thus, there exist a sufficiently large  $T_2 \geq t_0 \geq 0$  such that

$$x_2(t) < M_2 \text{ as } t > T_2. \tag{4}$$

From the fourth equation of the system (1), we can obtain

$$\dot{u}_1(t) \leq e_1^m - f_1^l u_1(t) + q_1^m M_1 = f_1^l \left[ \frac{e_1^m + q_1^m M_1}{f_1^l} - u_1(t) \right].$$

In view of the comparison theorem and the same analysis above, we can obtain that

(C) when  $0 < u_{10} < N_1^*$ , it holds that  $u_1(t) < N_1^* < N_1$  for  $t \geq t_0$ .

(D) when  $u_{10} \geq N_1^*$ , it holds that  $u_1(t) < N_1$  for a sufficiently  $t$ . Otherwise, if  $u_1(t) > N_1$ , then there exist  $\alpha > 0$  such that  $x_1(t) \geq M_1^* + \alpha$ . Moreover, we have

$$\dot{u}_1(t) \leq f_1^l \left[ \frac{e_1^m + q_1^m M_1}{f_1^l} - u_1(t) \right] \leq f_1^l [N_1^* - u_1(t)] < -\alpha f_1^l.$$

Integrating the above inequality yields

$$u_1(t) < u_1(t_0) - \alpha f_1^l (t - t_0).$$

Thus, it holds that  $u_1(t) < u_1(t_0) - \alpha f_1^l (t - t_0) \rightarrow -\infty$  as  $t \rightarrow +\infty$ . This is a contradiction, so there exist a sufficiently large  $T_4 \geq t_0 \geq 0$  such that

$$u_1(t) < N_1 \text{ as } t > T_4. \tag{5}$$

According to the fifth and sixth of system (1), we have

$$\dot{u}_2(t) \leq e_2^m - f_2^l u_2(t) + q_2^m M_2 = f_2^l \left[ \frac{e_2^m + q_2^m M_2}{f_2^l} - u_2(t) \right],$$

and

$$\dot{u}_3(t) \leq e_3^m - f_3^l u_3(t) = f_3^l \left[ \frac{e_3^m}{f_3^l} - u_3(t) \right].$$

Thus, employing the comparison theorem and the similar analysis which is used above, it holds that there exist sufficiently large  $T_5 \geq t_0 \geq 0$  and  $T_6 \geq t_0 \geq 0$ , such that

$$u_2(t) < N_2 \text{ as } t > T_5, \tag{6}$$

and

$$u_3(t) < N_3 \text{ as } t > T_6. \tag{7}$$

By the third equation of the system (1), we can get

$$\begin{aligned} \dot{x}_3(t) &= x_3(t)[-r_3(t) - a_3(t)x_3(t) + \frac{\mu_1(t)k_1(t)x_1^2(t)}{p_1(t) + x_1(t)} + \frac{\mu_2(t)k_2(t)x_2^2(t)}{p_2(t) + x_2(t)} + d_3(t)u_3(t)] \\ &\leq x_3(t)[-r_3^l - a_3^l x_3(t) + \frac{\mu_1^m k_1^m M_1^2}{p_1^l + M_1} + \frac{\mu_2^m k_2^m M_2^2}{p_2^l + M_2} + N_3 d_3^m] \\ &\leq x_3(t)[-r_3^l - a_3^l x_3(t) + \frac{\mu_1^m k_1^m M_1^2}{p_1^l} + \frac{\mu_2^m k_2^m M_2^2}{p_2^l} + N_3 d_3^m] \\ &\leq x_3(t) a_3^l [-x_3(t) + \frac{\mu_1^m k_1^m M_1^2 p_2^l + \mu_2^m k_2^m M_2^2 p_1^l + N_3 d_3^m p_1^l p_2^l - r_3^l p_1^l p_2^l}{a_3^l p_1^l p_2^l}]. \end{aligned}$$

Based on the comparison theorem, it follows that

(E) when  $0 < x_{30} < M_3^*$ , it holds that  $x_3(t) < M_3^* < M_3$ , for  $t \geq t_0$ .

(F) when  $x_{30} \geq M_3^*$ , it holds that  $x_3(t) < M_3$  for a sufficiently  $t$ .

Using the same analytical method as for the first equation, we obtain that there exists a sufficiently large  $T_3 \geq t_0 \geq 0$ , such that

$$x_3(t) < M_3, \text{ as } t > T_3. \tag{8}$$

On the other hand, by the same analysis process, we have

$$\begin{aligned} \dot{x}_1(t) &\geq x_1(t)[r_1^l - a_1^m x_1(t) - b_1^m M_2 - \frac{k_1^m M_3 x_1(t)}{p_1^l} - d_1^m N_1] \\ &\geq x_1(t)[- (a_1^m + \frac{k_1^m M_3}{p_1^l}) x_1(t) + r_1^l - b_1^m M_2 - d_1^m N_1] \\ &= \frac{a_1^m p_1^l + k_1^m M_3}{p_1^l} x_1(t) [-x_1(t) + \frac{p_1^l (r_1^l - d_1^m N_1 - b_1^m M_2)}{a_1^m p_1^l + k_1^m M_3}], \end{aligned}$$

Based on the comparison theorem, it follows that

(G) when  $m_1^* < x_{10}$  it holds that  $m_1 < m_1^* < x_1(t)$ , for  $t \geq t_0$ .

(H) when  $0 < x_{10} \leq m_1^*$ , it holds that  $m_1 < x_1(t)$ . for a sufficiently  $t$ . Otherwise, if  $u_1(t) > N_1$ , then there exist  $\beta > 0$  such that  $x_1(t) \geq m_1^* - \beta$ . Moreover, we have

$$\begin{aligned} x_1(t)|_{x_1(t) < m_1} &\geq \frac{a_1^m p_1^l + k_1^m M_3}{p_1^l} x_1(t) [-x_1(t) + \frac{p_1^l (r_1^l - d_1^m N_1 - b_1^m M_2)}{a_1^m p_1^l + k_1^m M_3}] \\ &\geq \frac{a_1^m p_1^l + k_1^m M_3}{p_1^l} x_1(t) [-x_1(t) + m_1^*] \\ &\geq \frac{a_1^m p_1^l + k_1^m M_3}{p_1^l} \beta x_1(t). \end{aligned}$$

Integrating the above inequality yields

$$x_1(t) \geq x_{10} \exp(\frac{\beta(a_1^m p_1^l + k_1^m M_3)}{p_1^l} (t - t_0)).$$

Thus, it holds that  $x_1(t) \geq x_{10} \exp(\frac{\beta(a_1^m p_1^l + k_1^m M_3)}{p_1^l} (t - t_0)) \rightarrow +\infty$  as  $t \rightarrow +\infty$ . This is a contradiction, so there exist a sufficiently large  $T_1' \geq t_0 \geq 0$ , such that

$$x_1(t) \geq m_1 \text{ as } t \geq T_1' . \tag{9}$$

Similarly, according to the second equation we get

$$\begin{aligned} \dot{x}_2(t) &\geq x_2(t)[r_2^l - a_2^m x_2(t) - b_2^m M_1 - \frac{k_2^m M_3 x_2(t)}{p_2^l} - d_2^m N_2] \\ &\geq x_2(t)[-(\frac{a_2^m p_2^l + k_2^m M_3}{p_2^l})x_2(t) + r_2^l - b_2^m M_1 - d_2^m N_2] \\ &= \frac{a_2^m p_2^l + k_2^m M_3}{p_2^l} x_2(t)[-x_2(t) + m_2^*]. \end{aligned}$$

Using the comparison theorem and the same analytical approach as above, we can obtain the existence of a sufficiently large  $T_2' \geq t_0 \geq 0$ , such that

$$x_2(t) > m_2 \text{ as } t \geq T_2' . \tag{10}$$

From the sixth equation of the system (1), we obtain

$$\dot{u}_3(t) = e_3(t) - f_3(t)u_3(t) - q_3(t)x_3(t) \geq f_3^m[-u_3(t) + \frac{e_3^l - q_3^m M_3}{f_3^m}].$$

In view of the comparison theorem and the same analysis above, we can obtain that

(I) when  $n_3 < u_{30}$  it holds that  $n_3 < n_3^* < u_3(t)$ , for  $t \geq t_0$ .

(J) when  $0 < u_{30} \leq n_3$ , it holds that  $n_3 < u_3(t)$  for a sufficiently  $t$ . Otherwise, if  $u_1(t) > N_1$ , then there exist  $\beta > 0$  such that  $u_3 \leq n_3^* - \beta$ . Moreover, we have

$$\dot{u}_3(t)|_{u_3(t) < n_3} \geq f_3^m[-u_3(t) + \frac{e_3^l - q_3^m M_3}{f_3^m}] = f_3^m[-u_3(t) + n_3^*] > f_3^m \beta.$$

So, it holds that  $u_3(t) > u_3(t_0) + f_3^m \beta(t - t_0) \rightarrow +\infty$ , as  $t \rightarrow +\infty$ . This is a contradiction, so there exist a sufficiently large  $T_4' \geq t_0 \geq 0$ , such that

$$u_3(t) \geq n_3 \text{ as } t \geq T_4' . \tag{11}$$

According to the fourth and fifth of system (1), we have

$$\dot{u}_1(t) \geq e_1^l - f_1^m u_1(t) + q_1^l m_1 \geq f_1^m[-u_1(t) + \frac{e_1^l + q_1^l m_1}{f_1^m}] = f_1^m[-u_1(t) + n_1^*],$$

and

$$\dot{u}_2(t) \geq e_2^l - f_2^m u_2(t) + q_2^l m_2 \geq f_2^m[-u_2(t) + \frac{e_2^l + q_2^l m_2}{f_2^m}] = f_2^m[-u_2(t) + n_2^*].$$

Thus, employing the comparison theorem and the similar analysis which is used above, it holds that there exist sufficiently large  $T_4' \geq t_0 \geq 0$  and  $T_5' \geq t_0 \geq 0$  such that

$$u_1(t) \geq n_1 \text{ as } t \geq T_4' , \tag{12}$$

and

$$u_2(t) \geq n_2 \text{ as } t \geq T_5' . \tag{13}$$

From the third equation of the system (1), we obtain

$$\begin{aligned} \dot{x}_3(t) &= x_3(t)[-r_3(t) - a_3(t)x_3(t) + \frac{\mu_1(t)k_1(t)x_1^2(t)}{p_1(t) + x_1(t)} + \frac{\mu_2(t)k_2(t)x_2^2(t)}{p_2(t) + x_2(t)} + d_3(t)u_3(t)] \\ &\geq x_3(t)[-r_3^m - a_3^m x_3(t) + \frac{\mu_1^l k_1^l m_1^2}{p_1^m + m_1} + \frac{\mu_2^l k_2^l m_2^2}{p_2^m + m_2} + d_3^l n_3] \\ &\geq x_3(t)[-r_3^m - a_3^m x_3(t) + \frac{\mu_1^l k_1^l m_1^2}{p_1^m + m_1} + \frac{\mu_2^l k_2^l m_2^2}{p_2^m + m_2}] \\ &= a_3^m x_3(t)[-x_3(t) + \frac{\mu_1^l k_1^l (p_2^m m_1^2 + m_1^2 m_2)}{a_3^m (p_1^m + m_1)(p_2^m + m_2)} + \frac{\mu_2^l k_2^l (p_1^m m_2^2 + m_1 m_2^2)}{a_3^m (p_1^m + m_1)(p_2^m + m_2)} \\ &\quad - \frac{-r_3^m (p_1^m p_2^m + p_1^m m_2 + p_2^m m_1 + m_1 m_2)}{a_3^m (p_1^m + m_1)(p_2^m + m_2)}] \\ &= x_3(t)a_3^m[-x_3(t) + m_3^*]. \end{aligned}$$

In view of the comparison theorem and the same analysis above, we can obtain that

(K) when  $m_3^* < x_{30}$  it holds that  $m_3 < m_3^* < x_3(t)$ , for  $t \geq t_0$ .

(L) when  $0 < x_{30} \leq m_3^*$ , it holds that  $m_3 < x_3(t)$  for a sufficiently large  $t$ . Otherwise, if  $m_3 > x_3(t)$ , then there exist  $\beta > 0$  such that  $x_3 \leq m_3^* - \beta$ . Moreover, we have

$$\dot{x}_3(t) \Big|_{x_3(t) < m_3} \geq x_3(t)a_3^m[-x_3(t) + m_3^*] > a_3^m \beta x_3(t),$$

it holds that  $x_3(t) > x_{30} \exp(a_3^m \beta(t - t_0)) \rightarrow +\infty$ , as  $t \rightarrow +\infty$ . This is a contradiction, so there exist a sufficiently large  $T_3' \geq t_0 \geq 0$ , such that

$$x_3(t) > m_3 \text{ as } t \geq T_3'. \tag{14}$$

From (3)-(14), and set  $T = \max_{1 \leq k \leq 6} \{T_k, T_k'\}$ , then we have  $m_i \leq x_i(t) \leq M_i, n_i \leq x_i(t) \leq N_i$ , as  $t > T$  for any positive solution  $(x_1(t), x_2(t), x_3(t), u_1(t), u_2(t), u_3(t))$  of the system (1) with the positive initial values. This end proof of **Theorem 1**.

## ACKNOWLEDGEMENTS

This work is supported by the Natural Science Foundation of Sichuan Province (Grant No. 2023NSFSC0071) and the National Natural Science Foundation (Grant No. 12101090) of China.

## REFERENCES

1. Lotka, A. J. Elements of Physical Biology, New York: Williams and Wilkins, 1925.
2. Vloterra, V. Variazionie fluttuazioni del numero d'individui in specie animali conviventi, Memorie deU'Accademia del Lincei, 2 (1926) 31-113.
3. Zhang, Z. Q. Z. C. Wang, Periodic Aolutions of a two-species ratio-dependent predator-prey system with time delay in a two-patch environment, Anziam Journal, 45 (2) (2003) 233-244.
4. Arditi, R. L. R. Ginzburg, Coupling in predator-prey dynamics: ratio-dependence, Journal of Theoretical Biology, 139 (1989) 311-326.
5. Abrams, P. A. The effects of ratio-dependent functional responses on the dynamics of predator-prey interactions, Ecology, 71 (3) (1990) 1026-1038.
6. Skalski, G. T. J. F. Gilliam, Functional responses with predator interference: viable alternatives to the Holling type II model, Ecology, 82 (11) (2001) 3083-3092.
7. Liao, X. Y. Z. G. Ouyang, S. F. Zhou, Permanence of species in nonautonomous discrete Lotka-Volterra competitive system with delays and feedback controls, Journal of Computational and Applied Mathematics, 211 (1) (2008) 1-10.

8. Wang, C. Y. Y. Q. Zhou, Y. H. Li, R. Li, Well-posedness of a ratio-dependent Lotka-Volterra system with feedback control, *Boundary Value Problems*, 2018 (2018), Article ID: 117.
9. Brown, J. S. B. P. Kotler, R. J. Smith, W. O. Wirtz II, The effects of owl predation on the foraging behavior of heteromyid rodents, *Oecologia*, 76 (1988) 408-415.

\*\*\*\*\*