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RESEARCH ARTICLE

THERMODYNAMICS OF THE VARIABLE CHAPLYGIN GAS MODEL IN BULK VISCOUS COSMOLOGY

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ABSTRACT

In this paper, we investigate the thermodynamic characteristics of a viscous fluid called viscous variable Chaplygin gas (VVCG) model. The equation of state $p = -\frac{B}{\rho}$ is used, with $B = B_0 V^{-\frac{n}{3}}$ and B_0, a positive universal constant. Our study has also taken consideration of the bulk viscosity coefficient, $\xi = \xi_0 \rho^{1/2}$. In order, to analyze the nature of the universe, the behavior of physical parameters in viscous fluid is discussed. The third law of thermodynamics validity is examined using the specific heat formalism. Furthermore, the thermal equation of state is discussed with the aid of thermodynamic entities. Adiabatic, specific heat, and isothermal conditions from classical thermodynamics are used to investigate thermodynamic stability.

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INTRODUCTION

The discovery of the cosmic acceleration of the universe presents a new challenge to fundamental physics and cosmology ideas (1–3). NASA's findings (5, 5) indicate that the mass of stars and galaxies is less than 5 percent of the universe's total mass. A few independent observations indicate that about 22 percent of the universe's total energy density is made up of cold dark matter particles that are not baryonic and clump together gravitationally but have never been directly observed. In contrast, about 73 percent is made up of gravitationally repulsive energy or a mystery dark energy. The universe's late acceleration is frequently attributed to the presence of dark energy, one of the most peculiar and mysterious elements, in the cosmic fluid. However, the nature of this dark energy is one of the most difficult theoretical issues in modern cosmology, and many people begin feverishly seeking for different viable explanations. In order, to spare readers from having to put up with the recurrence of those arguments and their opposites (for excellent evaluations in this area, (6, 7)), we overlook the details and note that numerous types of Chaplygin gas are also strong candidates for the same. Our primary aim is to ascertain whether the well-known stability criteria may impose stringent restrictions on the system's parameter values. The stability of the system is an important problem in this context, among many others. Several of the claims made by former employees are absolutely refuted when compared to the stability requirements. Considering this, we have already discussed the stability issue in several of our papers (8, 9), and this one carries on that conversation by concentrating on the VMCG model. Chaplygin-type gas cosmology (10, 11) suggests that a new matter field can be used to recreate recent occurrences related to dark energy. This type of equation of state (EOS) is not appropriate for the primordial universe (11, 12). This equation of state yields a component that operates as dust initially and thereafter as a cosmological constant (Λ). We are interested in the Chaplygin gas (CG) (13–18) as a potential dark energy candidate. To maintain agreement with observational data, the original model has been extended to the generalized Chaplygin gas (GCG) (19–23). Models like the modified Chaplygin gas (MCG) (24) and the modified cosmic Chaplygin gas (MCCG) (25) are the outcome of additional changes. Under some conditions, a successful dark energy model should be able to approximate the cosmological constant model while still being consistent with the data (26). The issue of late-time cosmic acceleration is effectively handled by the majority of Chaplygin gas-based models. However, they frequently fall short of resolving the initial singularity problem. The equation of state (EoS) for Chaplygin gas (CG) is as follows:

$$p = -\frac{B}{\rho} \tag{1}$$

where, B is a constant.

The scale factor of our selected measure determines the value of B, as per a recent presentation and constraint of a variable Chaplygin gas (VCG) model using SNeIa gold data (27, 28). We now understand the relationship above to be $B = B_0 V^{-\frac{n}{3}}$, where B_0 is a positive universal constant. When $n = 0$, the VCG equation of state reduces to the conventional Chaplygin gas equation of state. The value of n can be either positive or negative. A gold sample of 157 SNeIa data was utilized by Guo *et al.* (27) to show that $n = -3.4$ was the best fit value. In a different paper (28), they used the gold sample of 157 SNeIa data and the X-ray gas mass fraction in 26 galaxy clusters (29) to limit VCG and get the value that best matches $n = 0.5^{+1.0}_{-1.1}$. This result is consistent with a phantom-like Chaplygin gas model that permits a rise in dark energy density over time. Importantly, new evidence suggests that the severely negative state equation $\omega = -1$ (30–32) may suit the data quite well. However, another study (33) shows that the value of n is between (-1.3, 2.6) and (-0.2, 2.8). Santos et al. have investigated the thermodynamical stability of the GCG model (34). The thermodynamical behavior of the variable Chaplygin gas (VCG) will be investigated in this work, and we give both the integrability condition equation and the temperature of the equation. All thermal values are derived from temperature and volume. Here, we show that the third rule of thermodynamics is satisfied by the Chaplygin gas. Additionally, as a function of temperature or volume, we found a Chaplygin gas is appropriately represented by a novel universal equation of state. It is anticipated that the variable Chaplygin gas will behave similarly to the Chaplygin gas (CG). We thus confirm that CG might offer a coherent framework for comprehending dark matter and cooling energy during the expansion of the universe. Going back to the Chaplygin gas stability condition, we note that n should be negative. Surprisingly, $n = -3.4$ was the probability contour's best fit value, according to Guo et al. (27). It is conceivable for n to have a positive or negative best fit value, as was previously mentioned (33).

Furthermore, bulk viscosity has been demonstrated to be important in cosmology (35–37). The Chaplygin gas, which was first proposed in (38) and later confirmed by additional study in (39–44), may have hinted at the significance of viscosity early on. Refs. (37, 42) provide a detailed discussion of the VMCG and VMCCG models, which both take time-dependent energy density into account. Additionally, the VCG model is investigated in the framework of a non-flat FRW universe in Ref. (40). This idea offers a rational basis for understanding dark matter and dark energy. We study the combined effects of bulk viscosity and Chaplygin gas on a flat FRW universe. The conventional Friedmann equations, which are affected by the addition of bulk viscosity, are further influenced by the Chaplygin gas component. The model for the bulk viscous coefficient is $\xi = \xi_0 \rho^{1/2}$. as reported in Ref. (45). We use the thermodynamic criteria described in Ref. (46) to determine whether the conditions $\left(\frac{\partial p}{\partial V}\right)_S < 0$, $\left(\frac{\partial p}{\partial V}\right)_T < 0$, and $c_V > 0$ hold, following the methodology for the generalized in Santos *et.al.* (34) and modified Chaplygin gas models (47). These conditions must be satisfied to provide instantaneous thermodynamic stability. According to these references (34, 47, 48) and (9, 12), we investigate several cosmological parameters in the VVCG model, including the effective pressure p_{eff}, effective equation of state ω_{eff}, effective deceleration parameter q_{eff}, and the adiabatic square speed of sound as a function of volume and temperature. Ref. (49) further investigates the interaction between VVCG and $f(R, T)$ gravity in the FRW framework, where time-dependent contributions from the CG and dark energy in terms of pressure and energy density are incorporated into the modified Friedmann equations.

Basic Field Equations of FRW Cosmology

The following is the known expression for the Friedmann-Robertson-Walker metric equation:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \tag{2}$$

where, $d\Omega^2 = d\theta^2 + sin^2\theta d\varphi^2$, and $a(t)$ is the expansion rate of the universe. In addition to $k = 0, +1$, and -1 denoting flat, closed, and open universes, respectively, the dimensionless coordinates r, θ, and φ are also known as comoving coordinates. Since we are studying in flat spacetime in this instance, ($k = 0$), the Einstein equation can be expressed as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G T_{\mu\nu}}{c^4} \tag{3}$$

where $c = 1$, $8\pi G = 1$, and $\Lambda = 0$ are the parameters. We discovered that the universe's viscous variable Chaplygin gas (VVCG) functions as dark energy with EoS (1).

The following assertion is obtained with bulk viscous fluid by applying equations (2) and (3) (41, 43) as

$$T_{\mu\nu} = (\rho + p_{eff})u_\mu u_\nu - p_{\mu\nu} g_{\mu\nu} \tag{4}$$

where u^{μ} and ρ stand for the four-velocity vector and energy-density vector, respectively. Since, $T_{\mu\nu}$ & $g_{\mu\nu}$ are represents the energy-momentum tensor and metric tensor of the universe, respectively. Therefore, the total effective pressure of the viscous fluid can be written as

$$p_{eff} = p + \Pi = -\frac{B}{\rho} - \sqrt{3}\xi_0\rho = -\frac{B_0 V^{-\frac{n}{3}}}{\rho} - \sqrt{3}\xi_0\rho \tag{5}$$

where the bulk viscosity coefficient is $\xi = \xi_0\rho^{1/2}$, and the viscous pressure is $\Pi = -3\xi H$.
According to the definition, the energy-density is

$$\rho_{vvcg} = \frac{U}{V} \tag{6}$$

where the fluid's volume is denoted by V and its internal energy by U. We have the thermodynamics relation (50)

$$\left(\frac{\partial U}{\partial V}\right)_S = -p_{eff} \tag{7}$$

From equations (1), (5), (6) and (7), we get the relation

$$\left(\frac{\partial U}{\partial V}\right)_S = B_0\left(\frac{V^{1-\frac{n}{3}}}{U}\right) + \left(\sqrt{3}\xi_0\right)\frac{U}{V} \tag{8}$$

After integrating the previously described equation, the internal energy, U expression is

$$\boldsymbol{U} = \left(\frac{\boldsymbol{B_0}}{1-\sqrt{3\xi_0}-\frac{n}{3}}\boldsymbol{V}^{2-\frac{n}{3}} + \frac{\boldsymbol{b}}{\boldsymbol{V}^{(-2\sqrt{3}\xi_0)}}\right)^{\frac{1}{2}} \tag{9}$$

where the integration constant b is arbitrary.

The aforementioned expression can also be written as

$$U = \left(\frac{2B_0\,V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}} V\left(1 + \left(\frac{\epsilon}{V}\right)^{N}\right)^{\frac{1}{2}} \tag{10}$$

where,

$$\epsilon = \left[\frac{bN}{2B_0}\right]^{\frac{1}{N}} \tag{11}$$

and $N = 2\left(1 - \sqrt{3\xi_0} - \frac{n}{3}\right)$.

Consequently, another way to express the energy-density in the VVCG model is as

$$\rho_{vvcg} = \left(\frac{2B_0\,V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}}\left(1 + \left(\frac{\epsilon}{V}\right)^{N}\right)^{\frac{1}{2}} \tag{12}$$

Now we discuss the thermodynamical behaviour of this model.

Pressure

We calculate the VVCG model's effective pressure using the viscous parameter ξ_0 and volume V. Equations (5) and (12) are used to obtain

$$p_{eff} = \rho_{vvcg}\left[\left(-\sqrt{3\xi_0}\right) - \frac{N/2}{\left[1+\left(\frac{\epsilon}{V}\right)^{N}\right]}\right] \tag{13}$$

It can also be stated as

$$p_{eff} = -\left(\frac{2B_0 V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}} \frac{\frac{N}{2}}{\left[1+\left(\frac{\epsilon}{V}\right)^N\right]^{\frac{1}{2}}} \left[1 + \frac{\sqrt{3}\xi_0}{\frac{N}{2}}\left[1 + \left(\frac{\epsilon}{V}\right)^N\right]\right] \tag{14}$$

Effective pressure is expressed by the equation above. A negative total effective pressure results from the bulk viscous pressure phrase becoming more dominant and negative than the thermodynamic pressure. Dark energy is characterized by this "negative pressure" or tension, which propels the universe's observable faster expansion. Given the circumstances, we now obtain relations:

(a)For $n \neq 0$, and $\xi_0 = 0$, the above equation (14) gives to variable Chaplygin gas model (34)

$$p_{eff} = -\left(\frac{NB_0 V^{-\frac{n}{3}}}{2}\right)^{\frac{1}{2}} \left[1 + \left(\frac{\epsilon}{V}\right)^N\right]^{-\frac{1}{2}} \tag{15}$$

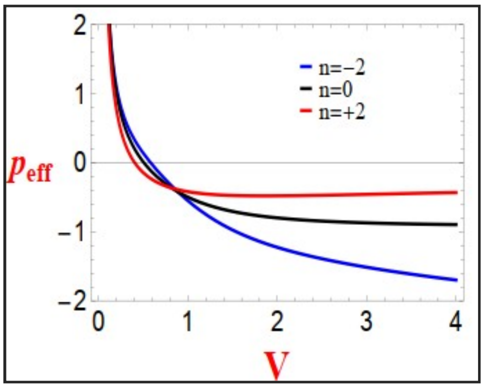

Fig. 1. The plot of p_{eff} vs. V for the values of $\xi_0 = -0.1$, $B_0 = 1$ & b = 1

The thermodynamic pressure is the same as the effective pressure. Both density and pressure typically fall as volume increases in an expanding universe.

For $\xi_0 = 0$ and $n = 0$, the results are reduced to the Chaplygin gas model (8) and previously, its thermodynamic behavior was examined by Santosh *et al.* (34)

$$p_{eff} = -B_0^{\frac{1}{2}} \left[1 + \left(\frac{\epsilon}{V}\right)^N\right]^{-\frac{1}{2}} \tag{16}$$

As seen in Fig.1, we plot effective pressure against volume for a particular value of viscous parameter ξ_0 and we have observed that the effective pressure decreases with volume as $n \leq 0$. Fig.1 shows that effective pressure is always negative for $n < 0$.

The critical volume (V_c) for the VVCG model is zero pressure condition, or $p_{eff} = 0$, is obtained as

$$V_c = \epsilon \left[\frac{(-\sqrt{3}\xi_0)}{1-\frac{n}{6}}\right]^{\frac{1}{N}} \tag{17}$$

Alternatively,

$$V_c = \left[\frac{bN(-\sqrt{3}\xi_0)}{2B_0\left(1-\frac{n}{6}\right)}\right]^{\frac{1}{N}} \tag{18}$$

A viable decelerated universe is indicated by a positive effective pressure (p_{eff}) when the magnitude of the critical volume is greater than volume V, i.e., $V_c > V$, and $p_{eff} = 0$ for $V = V_c$. Effective pressure turns negative when $V > V_c$, which further suggests that the cosmos is accelerating. In the analysis, we obtain a new scale of V_c when a dust-dominated universe enters the acceleration phase. We discover that ϱ and V_c are of the same order of magnitude. The decelerated universe is represented by $V << \epsilon$, whereas the very vast volume with a conceivable accelerated universe is represented by $V >> \epsilon$.

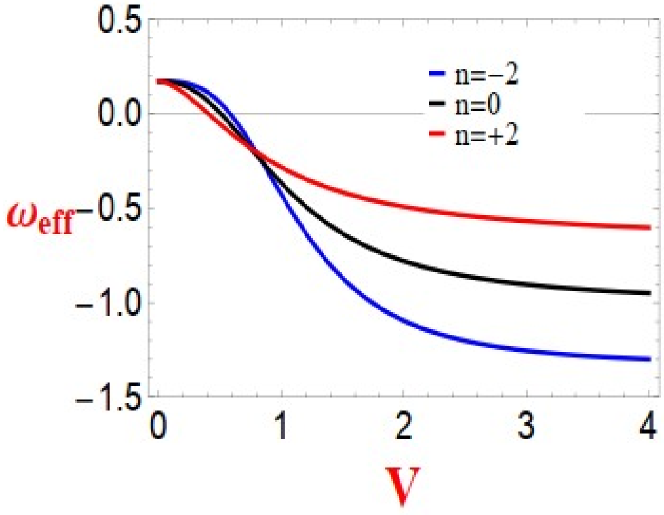

Fig. 2 The plot of ω_{eff} vs. V for the values of $\xi_0 = -0.1$, $B_0 = 1$ & b = 1.

Equation of State Parameter

The EoS for the bulk viscous is determined in this section and is defined as

$$\omega_{eff} = \frac{p_{eff}}{\rho_{vvcg}}. \tag{19}$$

Equations (12) and (14) can also be used to write the aforementioned expression as

$$\omega_{eff} = \left[\left(-\sqrt{3\xi_0}\right) - \frac{N/2}{\left[1+\left(\frac{\epsilon}{V}\right)^N\right]} \right] \tag{20}$$

In a viscous cosmology, the characteristics of the cosmic fluid and its bulk viscosity shape the dynamic relationship between the effective EoS parameter and volume. Instead of being constant, the effective EoS parameter changes when the cosmic scale factor (a) changes. The impact of bulk viscosity on the EoS is minimal during early eras when the cosmos is dense. The bulk viscosity may become dynamically significant as the universe gets denser and expands, pushing the effective EoS in the direction of negative values ($\omega_{eff} < —1/3$), which accelerates expansion as shown in Fig.2.

•When volume is small, $V << \epsilon$ *i.e.,* $\frac{\epsilon}{V} \gg 1$, the effective pressure is, $p_{eff} = (-\sqrt{3\xi_0})\, \rho_{vvcg}$, and $\omega_{eff} = -\sqrt{3\xi_0}$. It is observed that the effective pressure depends on viscous coefficient parameter ξ_0. If the value of ξ_0 is zero, $p_{eff} \approx 0$, the universe is dominated by dust, and EoS does not depend on n. If $\xi_0 > 0$, $\omega_{eff} <$ 1 which is indicated to a "Big Rip" scenario where expansion accelerates uncontrollably.

•For large volume, $V >> \epsilon$, *that is,* $\frac{\epsilon}{V} \ll 1$, and if $\xi_0 = 0$, then we get $\omega_{eff} \approx -1 + \frac{n}{6}$. If $n = 0$, $\omega_{eff} \approx —1$ which is leading to accelerated expansion i.e., Cold Dark Matter (ΛCDM). If n is positive value, since ω_{eff} will be $0 > \omega_{eff} > —1$, a big rip cannot occur and a quiescence phenomenon result. The phantom-like model, $\omega_{eff} < —1$, is obtained for $n < 0$. At $n = —2$ and $\xi_0 = —0.1$, ω_{eff} is more negative, as can be seen in Fig.2.

Deceleration Parameter: The deceleration parameter in a cosmology where a viscous fluid predominates dynamically changes with the universe's volume, which is represented by the scale factor. As a result of cosmic acceleration driven by viscosity, the deceleration parameter changes from a positive (decelerating) value in the early universe to a negative (accelerating) value in the later universe. In order, to explain the observable expansion history, this dynamic behavior offers an alternative to the cosmological constant (Λ). Radiation and matter densities were quite high in the early cosmos.

Based on the specification, the effective deceleration parameter that corresponds to the bulk viscosity of the VVCG model can be expressed as

$$q_{eff} = \frac{1}{2} + \frac{3}{2}\frac{p_{eff}}{\rho_{vvcg}} \tag{21}$$

The equation can be written using the viscous parameter ξ_0 using equation (20) as

$$q_{eff} = \frac{1}{2} + \frac{3}{2}\left[\left(-\sqrt{3\xi_0}\right) - \frac{N/2}{\left[1+\left(\frac{\epsilon}{V}\right)^N\right]} \right] \tag{22}$$

- The early universe, the volume is very small ($V << \epsilon$), the equation (22) reduces to $q_{eff} = \frac{1}{2}\left(1 - 3\sqrt{3\xi_0}\right)$ i.e., q_{eff} is positive for $\xi_0 \leq 0$, which indicates that the universe is decelerated. If viscosity coefficient (ξ_0) is zero, the deceleration parameter $q_{eff} = \frac{1}{2}$ i.e., the matter-dominated universe. Because of the large energy density and initial insignificant negative pressure associated with bulk viscosity, q_{eff} is positive, signifying a decelerating phase.
- The late universe, the volume is very large ($V >> \epsilon$), the equation (22) reduces to $q_{eff} = -1 + \frac{n}{4}$, also depends on n. When n is zero, $q_{eff} \approx -1$, universe is accelerated. Energy density falls as the universe gets bigger and its volume rises. However, the negative viscous pressure may start to dominate. As the cosmic volume expands, the negative bulk viscous pressure becomes more significant. In order, to move from a decelerating to an accelerating phase, the deceleration parameter falls, exceeds the threshold ($q_{eff} = 0, when\ n = 4$) (which corresponds to a constant expansion rate). In this instance, the *flip volume* (V_{eff}) will occur when the effective deceleration parameter is zero. In many viscous models, the deceleration parameter gets closer to $q_{eff} = -1$ as the scale factor gets closer to infinity. This is equivalent to a de Sitter universe, which behaves like a cosmological constant and experiences exponential growth driven by a constant vacuum energy density.

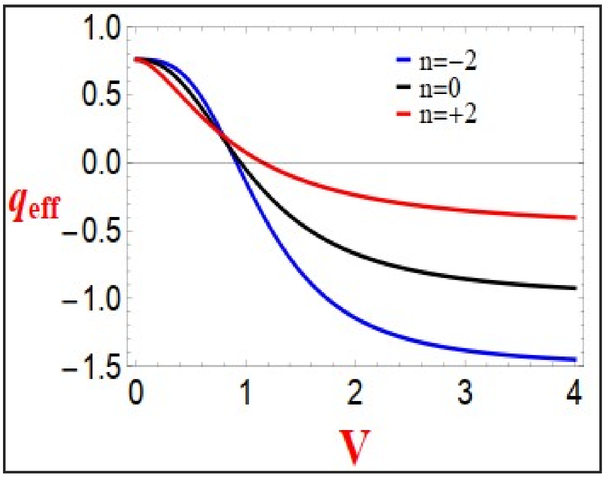

Fig. 3. The plot of q_{eff} vs. V for the values of $\xi_0 = -0.1$, $B_0 = 1$ & b = 1

Consequently, the flip volume expression can be stated as

$$V_f = \epsilon\left[\frac{2\left(1-3\sqrt{3\xi_0}\right)}{4-n}\right]^{\frac{1}{N}} = \left[\frac{bN\left(1-3\sqrt{3\xi_0}\right)}{B_0(4-n)}\right]^{\frac{1}{N}} \tag{23}$$

According to Fig.3, the cosmos accelerates as volumes rise after q_{eff} initially goes to zero. The equation above demonstrates that the value of V_f must be genuine when $\xi_0 < 0$; otherwise, there will not be any *flip*. Accordingly, the universe accelerates when $V > V_f$ and decelerates when $V < V_f$. Therefore, we determine that two scales of volume are V_c and V_f, respectively, from the zero-pressure condition, and that a potential deceleration-acceleration transition takes place at the effective deceleration parameter.

We now obtain the relation from equations (18) and (23) as

$$\frac{V_f}{V_c} = \left[\frac{\left(3\sqrt{3\xi_0}-1\right)(6-n)}{3\sqrt{3\xi_0}(4-n)}\right]^{\frac{1}{N}} \tag{24}$$

Square Speed of Sound: This analysis focused on the VVCG model's stability criteria. The definition of sound speed in a viscous fluid is

$$v_s^2 = \left(\frac{\partial p_{eff}}{\partial \rho_{vvcg}}\right)_S = \left[\left(-\sqrt{3\xi_0}\right) + \frac{N/2}{\left[1+\left(\frac{\epsilon}{V}\right)^N\right]}\right] \tag{25}$$

The velocity of sound in the cosmic fluid is a dynamically changing quantity that is governed by the bulk viscosity and thermodynamic parameters of the fluid rather than being a constant in viscous cosmology. The relation between sound speed and volume is driven by the expansion of the universe, as opposed to static fluids where density and elastic characteristics control sound speed. Sound speed is comparatively stable in the early cosmos because adiabatic processes dominate the fluid's characteristics. In the late universe, bulk viscosity increases with increasing volume, causing the sound speed to dynamically vary and possibly turn negative. This is a prerequisite for the fluid to function as a source of rapid expansion as shown in Fig.4. We also know that $0 < v_s^2 < 1$ must be the range of the speed of sound. This range was the focus of our investigation. It lowers to $v_s^2 = -\sqrt{3\xi_0}$ when volume is very small ($V << \epsilon$), i.e., at early universe. When volume is very large ($V >> \epsilon$), the equation (25) yields

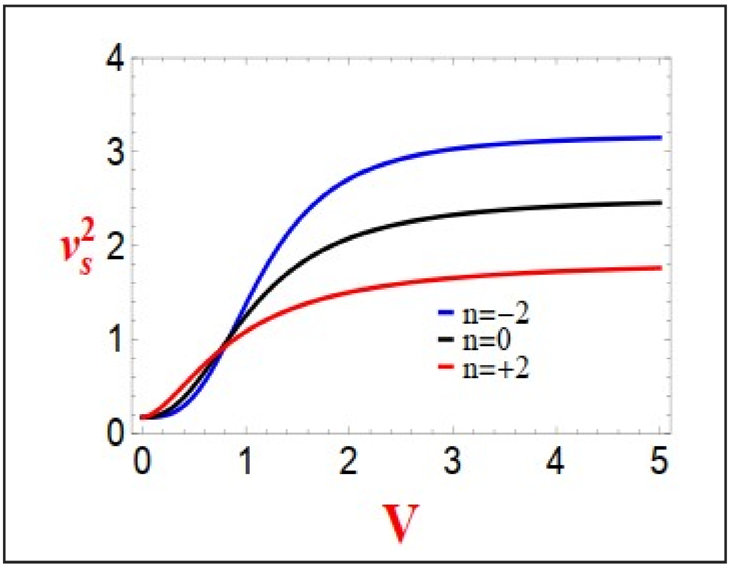

Fig. 4 The plot of v_s^2 vs. V for the values of $\xi_0 = -0.1$, B_0 = 1 & b = 1

$$v_s^2 = 1 - 2\sqrt{3\xi_0} - \frac{n}{6} \tag{26}$$

In the above equation, ξ_0 and n both are negative, $v_s^2 > 1$, i.e., the thermodynamical stability condition is satisfied, leading to a phantom type of universe (52, 53). If $\xi_0 > 0$ and $n > 0$, As seen in Fig.4, the equation (26) yields an imaginary speed of sound, which causes a perturbative cosmology (54). For holographic DE, Myung (55) discovered that always $v_s^2 < 0$, whereas for basic CG and tachyon matter, it is non-negative. According to Panigrahi and Chatterjee (56), with varying MCG, v_s^2 might be either positive or negative.

Thermodynamic Stability: We examine a fluid's thermodynamic stability conditions throughout the universe's evolution. The stability requirements of thermodynamics are known to be (46): In both adiabatic and isothermal expansion, the pressure decreases as (a) $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$, $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$ and (b) $c_V > 0$.

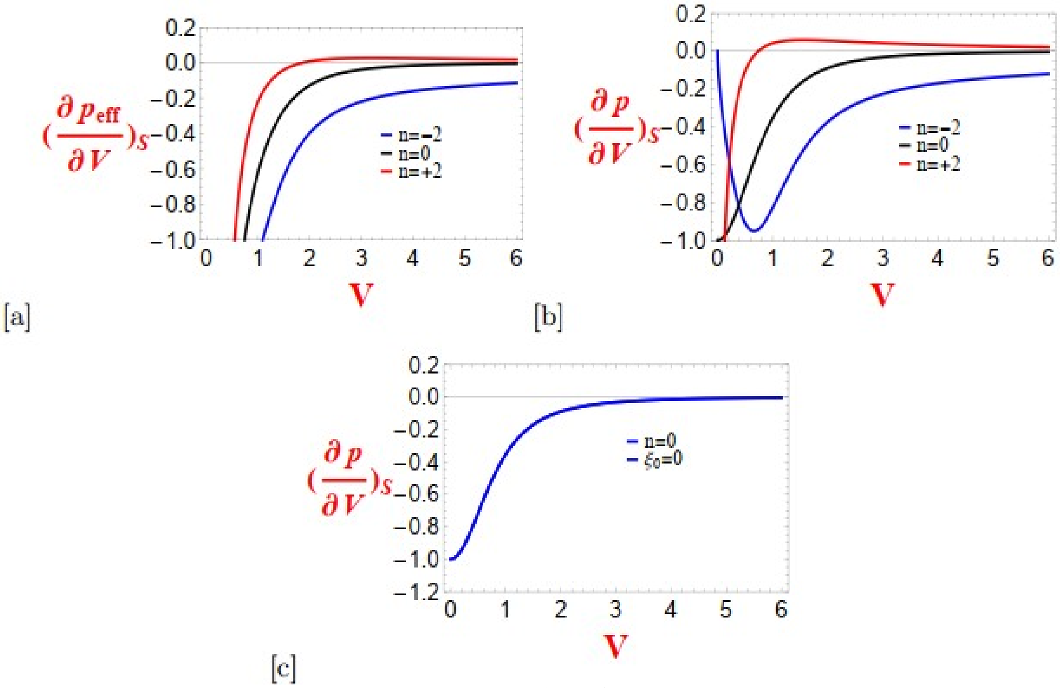

Fig. 5 The variation of effective pressure gradient $\left(\frac{\partial p_{eff}}{\partial V}\right)_V$ and volume V for values of $\xi_0 = -0.1$, $B_0 =$ 1, b = 1, and α = 0.5. Here, the graphs n = − 2 (blue line), n = 0 (black line) and $n = +2$ (red line); clearly shows that for throughout the evolution period.

Differentiating equation (14) w.r.t. volume, and the above equation can be expressed in terms pressure as

$$\left(\frac{\partial p_{eff}}{\partial V}\right)_S = -\frac{p_{eff}}{V\left[N+2\sqrt{3\xi_0}\left[1+\left(\frac{\epsilon}{V}\right)^N\right]\right]}\left[\frac{n\xi_0}{\sqrt{3}}\left[1+\left(\frac{\epsilon}{V}\right)^N\right]+N\left[\frac{n}{6}+\left(\frac{\epsilon}{V}\right)^N\left(\sqrt{3\xi_0}-\frac{N/2}{\left[1+\left(\frac{\epsilon}{V}\right)^N\right]}\right)\right]\right] \quad (27)$$

Firstly, when volume is very small (V << ϵ), the above expression can be written as $\left(\frac{\partial p_{eff}}{\partial V}\right)_S \approx -\frac{(1-\sqrt{3\xi_0})p_{eff}}{V}$. According to earlier research, in the early universe, $\left(\frac{\partial p_{eff}}{\partial V}\right)_S \approx (-\sqrt{3\xi_0})\rho_{vvcg}$, consequently, in this instance, $\left(\frac{\partial p_{eff}}{\partial V}\right)_S \approx -\frac{(1-\sqrt{3\xi_0})(-\sqrt{3\xi_0})\,\rho_{vvcg}}{V}$, this solely depends on ξ_0 and is independent of n. Hence, the pressure in this evolution is negative, therefore we can get $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$ for $\xi_0 < 0$. Secondly, the equation above (27) reduces to $\left(\frac{\partial p_{eff}}{\partial V}\right)_S \approx -\frac{np_{eff}}{6V}$ for large volumes ($V >> \epsilon$). In order, to make $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$, n must be negative because p_{eff} is negative at the late stage of evolution. The reliance of n is evident in the latter instance, as we have seen in Fig.5 of different Chaplygin gas models.

Fig. 5 The variation of effective pressure gradient $\left(\frac{\partial p_{eff}}{\partial V}\right)_V$ and volume V for values of $\xi_0 = -0.1$, $B_0 = 1$, b = 1, and α = 0.5. Here, the graphs n = − 2 (blue line), n = 0 (black line) and □ = +2 (red line); clearly shows that for throughout the evolution period.

The following scenarios will be discussed in order, to limit the parameters utilized here:

- If we put $\xi_0 = 0$ and $n \neq 0$, we find that equation (27) becomes a similar work of Panigrahi (8), we get

$$\left(\frac{\partial p_{eff}}{\partial V}\right)_S = \frac{p_{eff}}{6V}\left[\left[(6-n)\left(1-\frac{N/2}{\left[1+\left(\frac{\epsilon}{V}\right)^N\right]}\right)-n\right]\right] \quad (28)$$

From equation (28), we have seen that for $n \leq 0$, $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$ throughout the evolution. Fig.5 shows that the positive value of n is not compatible in VCG model. It is feasible to draw the conclusion that in order; to achieve thermodynamically stable development, the positive value of n needs to be removed. The characteristics of the graphs' evolution for $n = 0, n = -2$ and $n = +2$ are initially somewhat different, as Fig.5 illustrates, but $n = 0$ and $n = -2$ both yield $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$ throughout the evolution. The impact of n is the reason for this.

- If we put $\xi_0 = 0$ and $n = 0$, the equation (27) reduced to as given by

$$\left(\frac{\partial p_{eff}}{\partial V}\right)_S = \frac{p_{eff}}{V}\left(1-\frac{N/2}{\left[1+\left(\frac{\epsilon}{V}\right)^N\right]}\right) \quad (29)$$

which corresponds to Chaplygin gas. The graphical representation for $\xi_0 = 0$ and $n = 0$ (Chaplygin gas model), is shown in Fig.5 which shows that $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$ throughout the evolution and leading to the stability of fluid.

Thermal EoS: Using the thermodynamics relation, we also verified that the specific heat was positive at constant volume. Entropy and temperature can be used to express the specific heat as

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = V\left(\frac{\partial \rho_{vvcg}}{\partial T}\right)_V \quad (30)$$

where T and S are represents temperature and entropy, respectively. The equation is utilized to determine the fluid's temperature, $T = \left(\frac{\partial U}{\partial S}\right)_V$ (55), it can be expressed

$$T = \left(\frac{\partial U}{\partial b}\right)_V\left(\frac{\partial b}{\partial S}\right)_V \quad (31)$$

Using equation (**9**), the expression temperature can be written as

$$T = \frac{V^{(\sqrt{3\xi_0})}}{2}\left[\frac{2B_0V^N}{N}+b\right]^{\frac{1}{2}}\left(\frac{\partial b}{\partial S}\right)_V \quad (32)$$

Since b is a universal constant, we find that $\left(\frac{\partial b}{\partial s}\right)_V = 0$. This implies that, regardless of the pressure and volume, the temperature is zero. However, in the case of Chaplygin gas, the temperature does change with expansion. Therefore, we must consider $\frac{db}{dS} \neq 0$. Lacking specific information on how b depends on S, we assume that $\frac{db}{dS} > 0$ (8, 9), which allows us to derive positive temperatures that decrease through adiabatic expansion.

Equation (9) can be obtained by dimensional analysis

$$[b] = [U]^2\left[V^{(-\sqrt{3}\xi_0)}\right]^2 \tag{33}$$

Since we are aware that (U)=(T)(S), the equation above can be expressed as

$$[b] = [T]^2[S]^2\left[V^{(-2\sqrt{3}\xi_0)}\right] \tag{34}$$

It is challenging to solve for b from equation (34). Consequently, the expression for b in a trial case can be expressed as τ and υ

$$b = \left(\tau\upsilon^{-\sqrt{3}\xi_0}\right)^2 S^2 \tag{35}$$

In this case, □ represents the volume dimension and τ (the constant) represents merely the temperature. Presently, we get

$$\left(\frac{db}{dS}\right)_V = 2\left(\tau\upsilon^{-\sqrt{3}\xi_0}\right)^2 S \tag{36}$$

Using equations (35) and (36), the equation (32) becomes

$$T = V^{\left(1-\left(N+\frac{n}{3}\right)\right)}\left(\tau\upsilon^{-\sqrt{3}\xi_0}\right)^2 S\rho^{-1} \tag{37}$$

The above expression can be streamlined to

$$T = V^{(\sqrt{3}\xi_0)}\left(\tau\upsilon^{-\sqrt{3}\xi_0}\right)\left[1+\left(\frac{V}{\epsilon}\right)^N\right]^{-\frac{1}{2}} \tag{38}$$

Putting the value of b in equation (37), the entropy is given as

$$S(T) = \left(\frac{2B_0V^{-\frac{n}{3}}}{\mathrm{N}}\right)^{\frac{1}{2}} \frac{\left(\frac{V^{1-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)}{\left[\left(\frac{\tau\upsilon^{-\sqrt{3}\xi_0}}{TV^{-\sqrt{3}\xi_0}}\right)^2-1\right]^{\frac{1}{2}}} \tag{39}$$

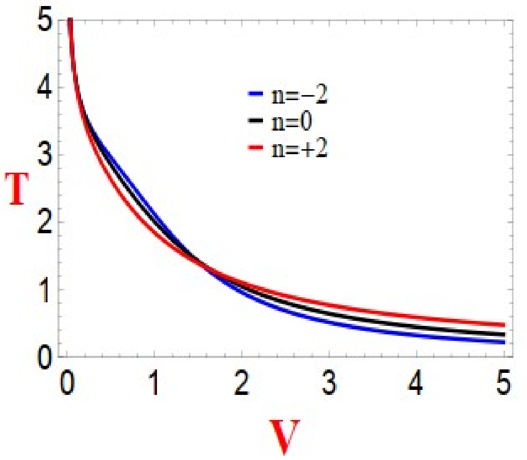

Fig. 6 The plot of T vs. V for the values of $\xi_0 = -0.1$, $B_0 = 1$ & b = 1

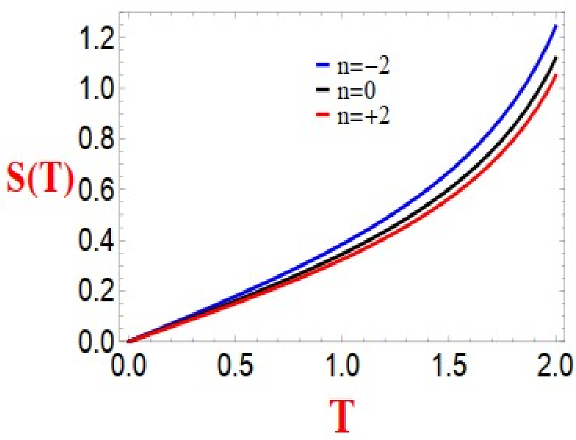

Fig. 7. The plot of S(T) vs. T for the values of $\xi_0 = -0.1$, $B_0 = 1$, V = 2, τ = 2.73, v = 1

Which means that is may also be written as

$$S = \left(\frac{2B_0}{\mathrm{N}}\right)^{\frac{1}{2}}\left(\frac{V^{1-\frac{n}{6}}}{T}\right)\frac{\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2}{\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]^{\frac{1}{2}}} \tag{40}$$

In contrast to the standard model, where temperature and volume are inversely related because of the universe's expansion, the VVCG model has a different relationship between temperature (T) and volume (V). This thermodynamic relation is altered by the introduction of a dissipative, heat-producing component in the form of a bulk viscous fluid. Fig.6 indicates that the temperature decreases to zero as the volume gets approach to the infinitely large value. The third law of thermodynamics is obeyed by the VVCG model in this regard. A perfect, non-viscous fluid expands adiabatically under a conventional cosmological model, which involves no heat exchange and constant entropy. When viscosity is included, the system generates entropy and becomes irreversible. However, the relationship between temperature and entropy is more complicated and relies on the cosmic epoch, whereas in a cosmological model with a viscous fluid, entropy continuously rises throughout time since viscosity is irreversible. Throughout cosmic history, cooling brought on by the universe's expansion has had varying effects on the evolution of entropy. In the early cosmos, viscosity was important, for example, in the quark-gluon plasma phase. According to causal thermodynamics-based models (such as the Israel-Stewart theory), the temperature may have risen exponentially during some stages, while viscous dissipation caused the comoving entropy to increase significantly. Fig.7, shows that the entropy increases with temperature in VVCG model.

From equation (40), we see that the condition we see that the condition $0 < TV^{-\sqrt{3}\xi_0} < \tau\upsilon^{-\sqrt{3}\xi_0}$ must hold for positive and finite entropy. This condition is confirmed as it meets the constraints $\tau > T > 0$, and $\upsilon < V < \infty$, where υ represents the lowest volume and τ represents the highest temperature. Therefore, the universe with VVCG represents a model of the universe that first decelerates and then accelerates. It is a thermodynamically stable system with $\xi_0 < 0$, positive squared velocity of sound, and positive heat capacity throughout its evolution. If $T \to 0$, then equation (40) yields $S = 0$, indicating that the thermodynamics' third law is satisfied. Substituting equations (38) and (40) into (30), it follows that

$$C_V(T) = T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{2B_0}{\mathrm{N}}\right)^{\frac{1}{2}}\left(V^{\frac{N}{2}}\right)\frac{\frac{TV^{-\sqrt{3}\xi_0}}{\left(\tau\upsilon^{-\sqrt{3}\xi_0}\right)^2}}{\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]^{\frac{1}{2}}} \tag{41}$$

In terms of entropy S, the above equation can also be written as

$$C_V(T) = \frac{S}{\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]} \tag{42}$$

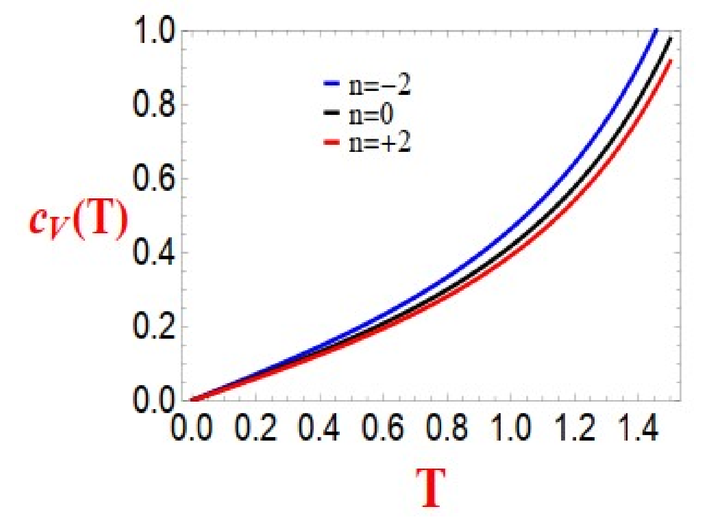

Fig. 8 The plot of c_V (T) vs. T for the values of $\xi_0 = -0.1$, B_0 = 1 V = 2, τ = 2.73, v = 1

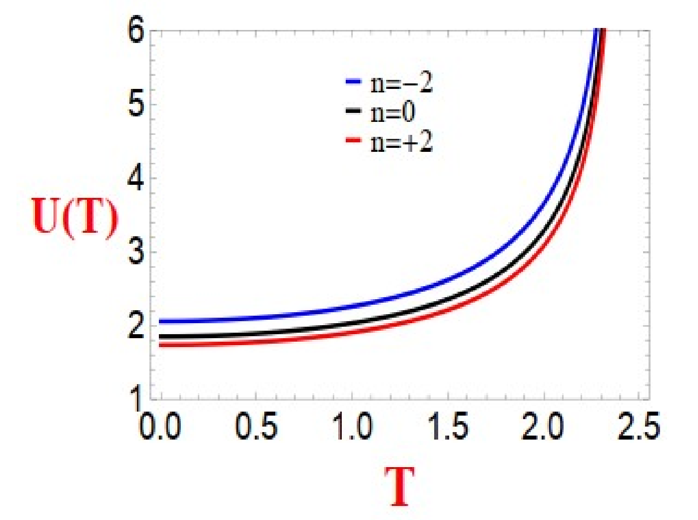

Fig. 9 The plot of U(T) vs. T for the values of $\xi_0 = -0.1$, B_0 = 1, V = 2, τ = 2.73, v = 1

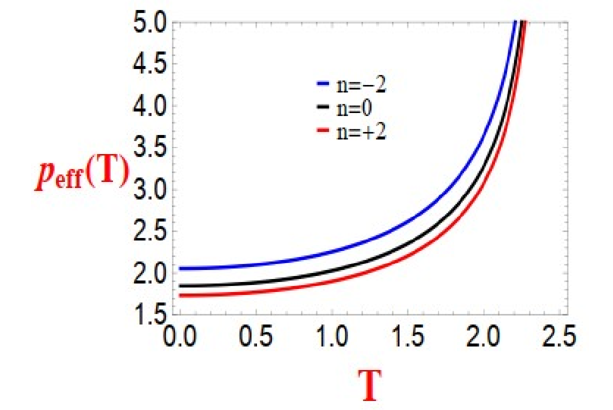

Fig. 10 The plot of p_{eff} (T) vs. T for the values of $\xi_0 = -0.1$, B_0 = 1, V = 2, τ = 2.73, v = 1

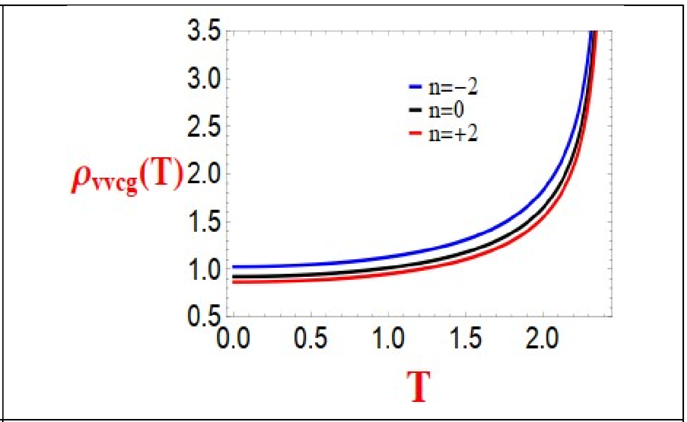

Fig. 11 The plot of ρ_{vvcg} (T) vs. T for the values of $\xi_0 = -0.1$, B_0 = 1, V = 2, τ = 2.73, v = 1

Thermal heat capacity in cosmic viscous fluids can change with temperature because of particle formation, non-equilibrium thermodynamics, and the early universe's relativistic character. Because of dissipative effects, viscous cosmic fluids differ from ideal fluids in that their heat capacity deviates from constant values, especially in the early universe when expansion rates and temperatures were extremely high. Since heat is not conserved in a comoving volume, the expansion of the universe is non-adiabatic due to the bulk viscosity in a Chaplygin gas. The cosmic expansion rate and the fluid's non-ideal fluid behaviour are thus closely related to the fluid's thermal characteristics.

When $0 < TV^{-\sqrt{3}\xi_0} < \tau\upsilon^{-\sqrt{3}\xi_0}$ i.e., $\tau > T > 0$ and $\upsilon < V < \infty$, and $\xi_0 < 0$, then C_V is positive i.e., $C_V > 0$, consistently satisfied, regardless of n's value. We found that in this model, the specific heat capacity vanishes at zero temperature, i.e., $C_V \to 0$, for $T \to 0$, verifies the third law of thermodynamics' validity. Thus, given the identical circumstance, we also saw that the thermal heat capacity C_V and the entropy S have positive values. Thus, for values of $\xi_0 < 0$ and $n < 0$ during the evolution, the VVCG model is thermodynamically stable. The plot of thermal heat capacity vs volume for a given value of $\xi_0 =$ — 0.1 and different values of n is displayed in Fig.8. In equation (42), if we enter $\xi_0 = 0$, we obtain an expression that is comparable to C_V, which was discovered by Panigrahi (8), and once more, we get the same expression for C_V for $n = 0$ that was discovered by Santosh *et al.* (34).

Using equations (9), (35) and (40) to calculate the internal energy of this model, we get

$$U(T) = \left(\frac{2B_0}{N}\right)^{\frac{1}{2}} \left(V^{1-\frac{n}{6}}\right) \left[1 - \left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]^{-\frac{1}{2}} \tag{43}$$

The internal energy of a viscous variable Chaplygin gas (VVCG) in a viscous fluid change with temperature because of the dissipative effects of bulk viscosity and an unusual equation of state of the gas. It differs from an ideal gas, where internal energy is just dependent on temperature, due to the complex connection that is dependent on the cosmological parameters and the viscosity's shape. As the cosmos expands, the fluid cools down and the internal energy becomes temperature-dependent in the stable models. Non-physical behaviours like endless temperatures or zero temperature states are avoided as a result. In contrast to a perfect VVCG, where the internal energy can be temperature-independent due to adiabatic evolution, bulk viscosity requires an explicit temperature dependence as shown in Fig.9. For thermodynamic stability, this is a prerequisite.

For the stability of the VVCG model, we looked at the isothermal condition, or $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$. Applying $p = p$ (*V, T)* from thermodynamic relations and resolving equations (14) and (35), we obtain

$$p_{eff}(T) = -\left(\frac{2B_0\, V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}} \left[1 - \left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]^{-\frac{1}{2}} \left[\sqrt{3\xi_0} + \frac{N}{2}\left[1 - \left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]\right] \tag{44}$$

The aforementioned statement can also be expressed in terms of entropy, therefore we

$$\boldsymbol{p_{eff}(T) = -\left(\frac{TS}{V}\right)\left(\frac{\tau\upsilon^{-\sqrt{3}\xi_0}}{TV^{-\sqrt{3}\xi_0}}\right)^2 \left[\sqrt{3\xi_0} + \frac{N}{2}\left[1 - \left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]\right]} \tag{45}$$

The density is expressed in temperature terms by

$$\boldsymbol{\rho_{vvcg}(T)} = \left(\frac{2B_0}{N}\right)^{\frac{1}{2}}\left(V^{-\frac{n}{6}}\right)\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2\right]^{-\frac{1}{2}} \tag{46}$$

In the viscous variable Chaplygin gas (VVCG) model, the variation of energy density with temperature is complex and dependent on the thermodynamic assumptions made. To derived thermal equation of state, and show that the fluid's energy density depends on temperature only under specific conditions. In Fig.11 shown that the relationship between energy density and temperature is proportional, meaning that it rises as the temperature does. A combination of its unique equation of state (EOS) and dissipative bulk viscosity causes pressure in a viscous fluid described by the viscous variable Chaplygin gas (VVCG) model to vary with temperature in a non-trivial manner. As the universe expands, energy is irreversibly dissipated into heat due to the bulk viscosity. This mechanism stops the expansion from being entirely adiabatic and influences the fluid's temperature evolution. The Hubble expansion rate and the bulk viscosity coefficient are included in the equation governing the temperature evolution, indicating that the fluid's temperature and pressure are inseparable due to cosmic dynamics. However, viscous theories have been put up to explain the universe's present fast expansion. The required negative pressure in this model to drive acceleration is provided by a bulk viscous pressure inside the component of dark energy as depicted in Fig.12. The variability of ρ_{vvcg} and p_{eff} when both T and V vary is plotted in Fig.12.

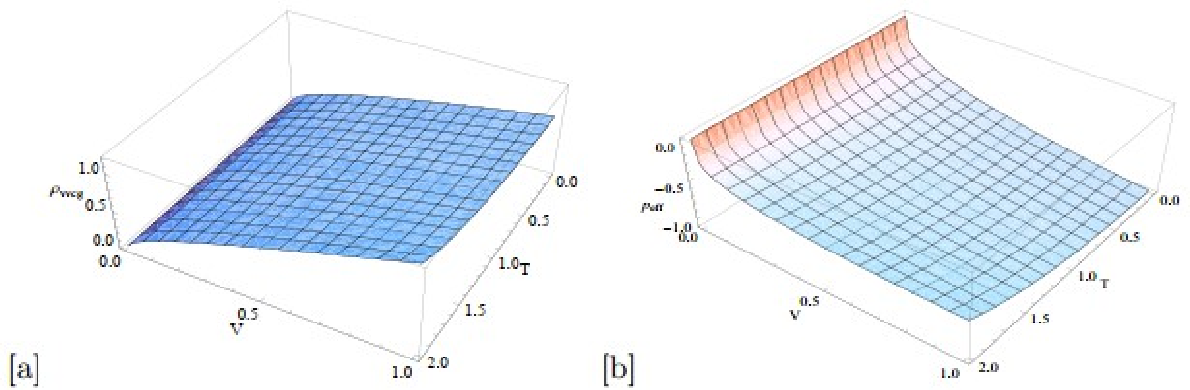

Fig. 12. The plot of ρ_{vvcg} and p_{eff} respectively when T and V vary with $\xi_0 = -0.1$, $B_0 = 1$, n = -2, τ = 2.73, v = 1 in VVCG model.

The form of the relevant thermal EoS parameter of VVCG is

$$\omega_{eff}(T) = -\left[\sqrt{3\xi_0} + \frac{N}{2}\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^2\right]\right] \tag{47}$$

A viscous fluid composed of a variable Chaplygin gas (VCG) has an equation of state (EOS) that changes with temperature due to dissipative bulk viscosity, in addition to simply relating pressure and energy density. This connection is constructed to provide thermodynamic stability, which is crucial for cosmological applications, rather than being assumed from the start. Both temperature and volume must be taken into account when determining the viscous VCG's pressure in order, to attain thermodynamic stability. This ensures the fluid's cooling as the universe expands, which is a prerequisite for a workable cosmological model. It has been found that the parameter in the equation of state is a clear function of temperature in viscous variable Chaplygin gas model as shown in Fig.13.

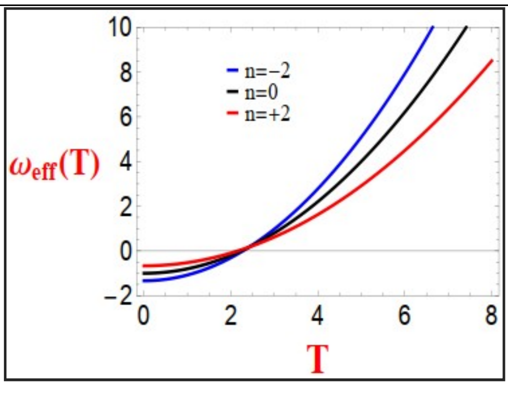

Fig. 13 The plot of $\omega_{eff}(T)$ vs. T for the values of $\xi_0 = -0.1$, $V = 2$, $\tau = 2.73$, $B_0 = 1$, $v = 1$

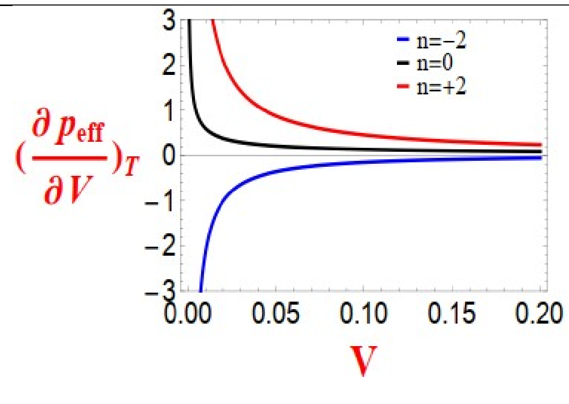

Fig. 14. The variation of effective pressure gradient $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ and volume V for values of ξ_0 = - 0.1, $B_0 = 1$, $\alpha = 0.5$, $T = 1$, $\tau = 2.73$ and $v = 1B_0 = 1$, $\alpha = 0.5$, $T = 1$, $\tau = 2.73$ and $v = 1$

Temperature (T) also affects the expression above. In the early universe, when temperatures were extremely high, $T \rightarrow \tau$, the equation (47) indicates that the cosmos is dominated by dust, as $\omega_{eff} = 0$ and $p_{eff} = 0$. The ΛCDM model is shown by the equation (47), which yields $\omega_{eff} = -1$ when the universe is in its late stage, when the temperature is extremely low, i.e., $T \rightarrow 0$.

We will now analyze equation (44) to see if $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$. Consequently, we obtain

$$\left(\frac{\partial \mathrm{p_{eff}}}{\partial \mathrm{V}}\right)_{\mathrm{T}} = -\frac{\left[V^{-1-\frac{n}{6}}\left(-\frac{B_0}{n+6\sqrt{3}\xi_0-6}\right)^2\right]\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau \upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]}{18\sqrt{2}\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau \upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]^{\frac{3}{2}}}$$

$$\mathrm{X}\left[\sqrt{3}n^2\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]-6\sqrt{3}n\left[1-\left(1-2\sqrt{3}\xi_0\right)\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]+108\xi_0\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\left[1-\sqrt{3}\xi_0-\frac{\sqrt{3}\xi_0}{\left[1-\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^2\right]}\right]\right] \quad (48)$$

In Fig.14 clearly indicates that $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$ for the negative values of n and ξ_0, throughout the evolution, which is thermodynamics stability condition satisfies. In this case, for $n < 0$ and $\xi_0 < 0$, the value of $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ should be negative, implying that the isobaric curves in our viscous variable Chaplygin gas (VVCG) model do not overlap with the isotherms within the thermodynamic state diagram. This distinction represents a meaningful advancement in our analysis. Since $n < 0$ and $\xi_0 < 0$, we infer that both $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$ and $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$ are negative, which is consistent with thermodynamic stability.

It is also possible to write the maximum temperature τ as a function of the expansion's starting circumstances. If the initial conditions at $V = V_0$ are $\rho = \rho_0, p_{eff} = p_0$, and $T = T_0$, we obtain equation (9)

$$b = V_0^{2(1-\sqrt{3}\xi_0)}\left(\rho_0^2 - 2B_0V_0^{-\frac{n}{3}}\right) \quad (49)$$

The energy density ρ and the effective pressure p_{eff} as a function of the volume V are obtained using equations (12), (13), and (49), we get

$$\rho = \left(\frac{2B_0\,V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}}\left[1+\frac{NV_0^{2(1-\sqrt{3}\xi_0)}}{2B_0V^N}\rho_0^2\left(1-\frac{2B_0\,V_0^{-\frac{n}{3}}}{N\rho_0^2}\right)\right]^{\frac{1}{2}} \quad (50)$$

and

$$p_{eff} = -\left(\frac{2B_0\,V^{-\frac{n}{3}}}{N}\right)^{\frac{1}{2}}\left[1+\frac{NV_0^{2(1-\sqrt{3}\xi_0)}}{2B_0V^N}\rho_0^2\left(1-\frac{2B_0\,V_0^{-\frac{n}{3}}}{N\rho_0^2}\right)\right]^{\frac{1}{2}}\left[\sqrt{3}\xi_0+\frac{N}{2}\left[1+\frac{NV_0^{2(1-\sqrt{3}\xi_0)}}{2B_0V^N}\rho_0^2\left(1-\frac{2B_0\,V_0^{-\frac{n}{3}}}{N\rho_0^2}\right)\right]^{-1}\right] \quad (51)$$

Equations (44), (50), and (51) may now be expressed as functions of the reduced parameters η, v, P, κ, t, and τ^* so that

$$\eta = \frac{\rho}{\rho_0}, v = \frac{V}{V_0}, P = \frac{p_{eff}}{B_0^2}, \kappa = \frac{2B_0}{N\rho_0^2}, t = \frac{T}{T_0}, \tau^* = \frac{\tau}{T_0} \quad (52)$$

It is now possible to represent the equations (44), (50), and (51) in the reduced units as

$$P = -\frac{\left(\frac{N}{2}\right)^2 v^{-\frac{n}{6}}\left[2\sqrt{3}\xi_0+1-\frac{1}{(\tau^*)^2}v^{-2\sqrt{3}\xi_0}\right]}{\left[1-\frac{1}{(\tau^*)^2}v^{-2\sqrt{3}\xi_0}\right]^{\frac{1}{2}}} \quad (53)$$

$$\eta = \kappa^{\frac{1}{2}} V^{-\frac{n}{6}} \left[1 + \frac{\left(1-\kappa V^{-\frac{n}{3}}\right) V^{\frac{n}{3}}}{\kappa V^{2(1-\sqrt{3}\xi_0)}}\right]^{\frac{1}{2}} \tag{54}$$

$$P = -\left(\frac{N}{2}\right)^{\frac{1}{2}} V^{-\frac{n}{6}} \left[\frac{2\sqrt{3}\xi_0}{N} + \frac{1}{1+\frac{\left(1-\kappa V^{-\frac{n}{3}}\right) V^{\frac{n}{3}}}{\kappa V^{2(1-\sqrt{3}\xi_0)}}}\right] \left[1 + \frac{\left(1-\kappa V^{-\frac{n}{3}}\right) V^{\frac{n}{3}}}{\kappa V^{2(1-\sqrt{3}\xi_0)}}\right]^{\frac{1}{2}} \tag{55}$$

At $P = P_0$, $V = V_0$ and $T = T_0$, we have $t = 1$ and $v = 1$ and we get from equations (53) and (55),

$$P_0 = \left(\frac{N}{2}\right)^{\frac{1}{2}} V_0^{-\frac{n}{6}} \left[\frac{2\sqrt{3}\xi_0 V_0^{\frac{n}{6}}}{N\kappa^{\frac{1}{2}}} + \frac{\kappa^{\frac{1}{2}}}{V_0^{\frac{n}{6}}}\right] = \left(\frac{N}{2}\right)^{\frac{1}{2}} V_0^{-\frac{n}{6}} \left[\frac{\frac{2\sqrt{3}\xi_0}{N}}{\left[1-\frac{1}{(\tau^*)^2} V_0^{-2\sqrt{3}\xi_0}\right]^{\frac{1}{2}}} + \left[1-\frac{1}{(\tau^*)^2} V_0^{-2\sqrt{3}\xi_0}\right]^{\frac{1}{2}}\right] \tag{56}$$

Thus, κ and τ^* can be calculated as follows:

$$\kappa = V_0^{\frac{n}{3}} \left(1 - \frac{1}{(\tau^*)^2} V_0^{-2\sqrt{3}\xi_0}\right) \tag{57}$$

and

$$\tau^* = \frac{V_0^{-\sqrt{3}\xi_0}}{\left(1-\kappa V_0^{-\frac{n}{3}}\right)^{\frac{1}{2}}} \tag{58}$$

We have observed that τ^* depends on ξ_0, κ, V_0, and n. All the aforementioned equations reduce to the equations of Panigrahi (8) for $\xi_0 = 0$, and to the equations of Santos *et al.* (34) for $n = 0$ and $\xi_0 = 0$. Since $\kappa = \frac{2B_0}{N\rho_0^2}$ at the present epoch, $\rho_0 = \left(\frac{2B_0}{\kappa N}\right)^{\frac{1}{2}}$. Considering the Planck era temperature $\tau = 10^{32} K$ and the present epoch temperature $T_0 = 2.7K$, the ratio $\tau^* = \frac{\tau}{T_0} = 3.7 \times 10^{31}$. Thus, the ratio κ will be

$$\boldsymbol{\kappa = V_0^{\frac{n}{3}} \left(1 - \frac{1}{(3.7\times 10^{31})^2} V_0^{-2\sqrt{3}\xi_0}\right)} \tag{59}$$

Once more, using equation (46), for the present epoch when T is extremely low (i.e., $T \to 0$),

$$\rho_0 = \left(\frac{2B_0}{N V_0^{\frac{n}{3}}}\right)^{\frac{1}{2}} \approx \left(\frac{2B_0}{\kappa N}\right)^{\frac{1}{2}}. \tag{60}$$

For large volumes, the equation (12) yields the same conclusion. Therefore, according to equation (35), the energy density ρ of the universe filled with VVCG at this epoch must be very close to $\left(\frac{2B_0}{\kappa N}\right)^{\frac{1}{2}}$.

CONCLUSIONS

We have examined the thermodynamic behavior of the VVCG model. We analyze the value of $n = -2, 0, +2$ that was found by Panigrahi & Chatterjee (56). Prior studies have demonstrated that n can be either positive or negative (33). Even while a detailed analysis of the most recent cosmological findings suggests that dark energy of some sort exists in the universe, it is very difficult to compare the benefits of its different forms, at least according to the observational data. In actuality, most of the options are within the energy budget. As a result, we have begun examining whether the gas in the equation operates as a thermodynamically closed system with suitable parameter values that meet the observational requirements for the specific instance of different types of Chaplygin gases. In this instance, the stability requirements for the gas have verified its consistency, and the conventional guideline has been followed: $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$, $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$, and $c_V > 0$. In order, to satisfy the third law of thermodynamics, the viscous variable Chaplygin gas is demonstrated here. Furthermore, we have found that temperature is the only factor influencing all thermal quantities, completely describing the viscosity variable Chaplygin gas with the value $\xi_0 = -0.1$. Furthermore, we find a new effective equation of state, deceleration parameter, and squared speed of sound that completely defines the viscous variable Chaplygin gas as a function of temperature and volume. Interestingly, this necessitates that the extra parameter n that is added to VVCG be negative definite. This result is consistent with Sethi *et al.* (33), who discovered that the range of $(-1.3, 2.6)$ contains n's best fit value. But according to Lu *et al.* (57), $n > 0$. This conclusion is dubious based on our findings because Lu's model becomes thermodynamically unstable in such a scenario.

- we have demonstrated that our model causes the pressure to become increasingly negative as volume grows for $\xi_0 < 0$ for n values that are negative, as shown in Fig.1.
- Furthermore, our model shows that the EoS increases to $p_{eff} = 0$ in a dust-dominated universe. For $n = 0$, the ΛCDM model is obtained when $\omega_{eff} \approx -1 + \frac{n}{6}$. According to the thermodynamical stability conditions, a phantom model, and the associated huge rip problem, $n < 0$ is shown to be beneficial. The situation described above is depicted in Fig.2. However, it is eventually discovered that the phantom-like evolution is consistent with SNeIa observations and CMB anisotropy measurements (52, 53).
- We have also explored the deceleration parameter in relation with thermo- dynamics, as seen in Fig.3. Our computation of the flip volume shows that the flip is possible for $n < 4$. For a dust-dominated universe, we get $q_{eff} = \frac{1}{2}$, whereas the universe accelerates at a late stage when $q_{eff} < 0$ for $n < 0$. This is consistent with the results of the observational research (27).
- We have also studied the square speed of sound in terms of the viscous parameters ξ_0 and n in the cosmic fluid of thermodynamics, as shown in Fig.4. When both ξ_0 and n are negative, the thermodynamical stability criterion is satisfied by the positive squared speed of sound (52, 53), resulting in a phantom-type world.
- We are mostly interested in the thermodynamic stability of the chosen gas. Our study demonstrates that $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$ throughout the evolution, which is also a stability condition, only for the negative value of n, as shown in Fig.5. In this case, the thermal heat capacity (c_V) will be computed because c_V is always positive for n < 0. As a result, both viscous fluid's thermodynamic stability criteria are investigated, showing that the fluid retains its thermodynamic stability throughout the evolution process.
- Equations (40) shows the expressions of temperature and entropy that we have determined. The third rule of thermodynamics clearly states that both $S = 0$ and c_V of the VVCG model vanish at $T = 0$. We also looked at the thermal and caloric EoS, both of which demonstrate that $0 < T < \tau$.
- We have also computed all thermal values with respect to temperature. Temperature is the only explicit determinant of the thermal EoS feature. Volume increases with temperature when $\xi_0 < 0$ experiences adiabatic expansions. In this case, the equation (47) simplified to $\omega_{eff} = 0$ at large temperature, $T \to \tau$ indicates that dust dominates the universe. When the temperature is very low, i.e., $T \to 0$, the ΛCDM model as depicted in *Fig.* is obtained from the equation (47), which produces $\omega_{eff} = -1$.
- As demonstrated in Fig.14, equation (48) demonstrates that $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$ during the evolution for $n < 0$ and $\xi_0 < 0$. This is a stability requirement. In contrast, our viscous variable Chaplygin gas (VVCG) model's isobaric curves naturally become negative for $n < 0$, demonstrating that the thermodynamic stability criteria are satisfied.

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Declarations: Conflicts of Interest The authors declare that they have no competing interests.

REFERENCES

1. Riess, A G. *et al.*, *Astron. J.*, 116, 1009, (1998)
2. Knop, R A. *et al.*, *Astrophys. J.*, 598, 102, (2003)
3. Amanullah, R *et al.*, *Astrophys. J.*, 598, 102, (2003)
4. Spergel, D N. *et al.*, *Astrophys. J. Suppl.*, 148, 175, (2003)
5. Spergel, D N. *et al.*, *Astrophys. J. Suppl.*, 170, 377, (2007)
6. Neupane I P. and H Trowland, *Int. J. Mod. Phys. D*, 19, 364, (2010)
7. Sahni V. and A Starobinsky, *Int. J. Mod. Phys. D*, 15, 2105, (2006)
8. Panigrahi, D. *Int. J. Mod. Phys. D*, 24, 1550030, (2015)
9. Panigrahi D. and S Chatterjee, *JCAP*, 1605, 052, (2016)
10. Kamenshchik, A Y U. Moschella and V. Pasquier, *Phys. Lett. B*, 511, 265, (2001)
11. Bento, M C O. Bertolami and A A Sen, *Rev. Phys. D*, 66, 043507, (2002)
12. Panigrahi D. and S Chatterjee, *JCAP*, 10, 002, (2011)

13. Makler, M. *et al.*, *Phys. Lett. B*, 555, 1, (2003)
14. Sandvik, H. *et al.*, The end of unified dark matter? *Rev. Phys. D*, 69, 123524, (2004)
15. Zhu, Z H. *Astron. Astrophys. J.*, 423, 421, (2004)
16. Wang, Y D. Wands, L Xu, J De-Santiago and A Hojjati, *Phys. Rev. D*, 87, 083503, (2013)
17. Setare, M R. *Phys. Lett. B*, 648, 329, (2007)
18. Setare, M R. *Int. J. Mod. Phys. D*, 18, 419, (2009)
19. Bilic, N G B. Tupper and R D Viollier, *Phys. Lett. B*, 535, 17, (2002)
20. Bazeia, D. *Phys. Rev. D*, 59, 085007, (1999)
21. Xu, L. J Lu and Y Wang, *Eur. Phys. J. C*, 72, 1883, (2012)
22. M R Setare, *Phys. Lett. B*, 654, 1, (2007)
23. M R Setare, *Eur. Phys. J. C*, 52, 689, (2007)
24. U Debnath, A Banerjee and S Chakraborty, *Class. Quantum Gravity*, 21, 5609, (2004)
25. H Saadat and B Pourhassan, *Astrophys. Space Sci.*, 344, 237, (2013)
26. H K Jassal, J S Bagla and T Padmanabhan, *Mon. Not. Roy. Astron. Soc.* 405, 2639, (2010)
27. Z K Guo, and Y Z Zhang, *Phys. Lett. B*, 645, 326, (2007)
28. Z K Guo, and Y Z Zhang, arXiv: astro-ph/0509790
29. S W Allen, *et al.*, *Mon. Not. R. Astron. Soc.*, 353, 457, (2004)
30. G R Bengochea, *Phys. Lett. B*, 695, 405, (2011)
31. B Novosyadlyj, O Sergijenko, R Durrer and V Pelykh, *Phys. Rev. D*, 86, 083008, (2012)
32. A V Astashenok, S Nojiri, S D Odintsov and A V Yorov, *Phys. Lett. B*, 709, 396, (2012)
33. G. Sethi, S K Singh and P Kumar, *Int. J. Mod. Phys. D*, 15, 1089, (2006)
34. F C Santos, M L Bedran and V Soares, *Phys. Lett. B*, 636, 86, (2006)
35. H Saadat, *Int. J. Theor. Phys.*, 51, 1317, (2012)
36. I Brevik and S.D. Odintsov, *Phys. Rev. D*, 65, 067302, (2002)
37. I Brevik, E Elizalde, S Nojiri and S D Odintsov, *Phys. Rev. D*, 84, 103508, (2011)
38. X H Zhai, *et al.*, arXiv:astro-ph/0511814
39. H Saadat and B. Pourhassan, *Astrophys. Space Sci.*, 343, 783, (2013)
40. Y D Xu, *et al.*, *Astrophys. Space Sci.*, 337, 493, (2012)
41. H Saadat and H Farahani, *Int. J. Theor. Phys.*, 52, 1160, (2013)
42. A R Amani and B Pourhassan, *Int. J. Theor. Phys.*, 52, 1309, (2013)
43. B Pourhassan, *Int. J. Mod. Phys. D* 22, 1350061, (2013)
44. J Sadeghi and H Farahani, *Astrophys. Space Sci.*, 347, 209, (2013)
45. P Thakur, *Int. J. Phys.*, DOI:10.1007/s12648-017-1126-8
46. L D Landau and E M Lifschitz, Statistical Physics, *Course of Theoretical Physics*, Vol.5, (Butterworth-Heinemann, London, 1984)
47. F C Santos, M L Bedran and V Soares, *Phys. Lett. B*, 646, 215, (2007)
48. M L Berdran and V Soares, *Prog. Theor. Physics*, 123, 51, (2010)
49. E H Baffou, M J S Houndjo and I G Salako, *Int. J. Geo. Met. Mod. Phys.*, 14, 1750051, (2017)
50. F Reif, *Fundamentals of Statistical and Thermal Physics*, Published by Levant Books, (Kolkata, 161, 2010)
51. A Kamenschick, U Moschella and V. Pasquier, *Phys. Lett. B*, 511, 265, (2001)
52. L P Cimento and R Lazkov, *Phys. Rev. Lett.*, 91, 211301, (2003)
53. C Kaconikhon, B Gumjudpai and E N Saridakis, *Phys. Lett. B*, 695, 10, (2011)
54. J C Fabris and J Martin, *Phys. Rev. D*, 55, 5205, (1997)
55. Y S Myung, *Phys. Lett. B*, 652, 223, (2007)
56. D Panigrahi and S Chatterjee, JCAP, doi:10.1088/1475-7516/2016/05/052
57. J Lu, *Phys. Lett. B*, 680, 404, (2009)

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