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RESEARCH ARTICLE

EVALUATING A POSTERIORI GEOMETRIC HYPOTHESES IN SPATIAL DATA: CONSTRAINED LOGARITHMIC CURVE PATTERNS IN A SUMMIT LANDSCAPE

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ABSTRACT

Geometric patterns identified in spatial data after inspection are difficult to evaluate statistically. When hypotheses are formulated *a posteriori*, conventional tests can overestimate significance because exploratory choices are not accounted for. This problem is pronounced in small-N spatial point sets, where model flexibility and feature selection strongly influence outcomes.A constrained evaluation framework is applied to assess *a posteriori* geometric hypotheses in spatial data. The approach limits the geometric degrees of freedom and conditions tests on a fixed set of candidate points. It is intended for situations in which a geometric pattern is first observed and then formally assessed. Point-to-curve deviations are used to compare the observed configuration with alternative spatial and geometric arrangements subject to specified constraints.The framework is demonstrated using a summit landscape in Central Bosnia, where a constrained logarithmic curve pattern has been proposed to link a small set of named summit locations derived from LiDAR data. The observed configuration occupies an extreme position relative to alternative constrained configurations within the defined summit set.The analysis is limited to spatial geometry and does not address origin or interpretation. The contribution is a transparent method for evaluating *a posteriori* geometric hypotheses in small-N spatial datasets.

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INTRODUCTION

***A posteriori* geometric hypotheses in spatial data:** Geometric patterns in spatial data are often identified through visual inspection. Curves, alignments, and regular forms are easy to perceive, even in sparse point sets. This sensitivity creates a methodological problem when a geometric hypothesis is formulated after the data has been observed. In such cases, standard statistical tests may give misleading results because they do not account for exploratory choices made during pattern recognition. This issue has been discussed widely in statistics and spatial analysis. Exploratory analysis can lead to implicit selection of features, parameters, and spatial extents that favour an apparent pattern. When these choices are not incorporated into the testing framework, reported significance can be inflated. This concern is not limited to formal hypothesis testing but applies more broadly to any attempt to assess non-random structure in spatial data (Ripley 1977; Fortin and Dale 2005; Gelman and Loken 2014). The problem is amplified in small-N spatial datasets. When the number of points is limited, geometric flexibility increases. Many curve families can be adjusted to pass near a small set of locations, especially when anchoring, orientation, and scale are free to vary. Visual correspondence alone is therefore insufficient to establish that a detected pattern reflects meaningful spatial structure rather than accommodation by a flexible geometry. These challenges do not imply that geometric hypotheses should be excluded from spatial analysis. Instead, they highlight the need for evaluation strategies that explicitly recognizethe *a posteriori* nature of such hypotheses and restrict the degrees of freedom available during testing.

Constraints, conditioning, and small-N spatial testing: A common response to pattern-detection problems is the use of randomisation or Monte Carlo simulation. In spatial contexts, this often involves comparing an observed configuration to patterns generated under uniform random placement. While useful as a baseline, unconstrained random models are frequently inappropriate for real landscapes. Spatial points are shaped by terrain, accessibility, and other structural factors that uniform models ignore. Conditioned approaches provide an alternative. Rather than generating entirely new point patterns, tests can be restricted to selections or rearrangements that preserve key features of the observed spatial structure. Such approaches are common in spatial ecology and point-pattern analysis, where null models are designed to reflect

known constraints rather than abstract randomness (Besag and Diggle 1977; Gotelli 2000; Baddeley et al. 2015). Small-N settings allow further refinement. When the candidate set of spatial features is limited and clearly defined, exhaustive evaluation becomes possible. Instead of estimating probabilities through repeated simulation, all admissible configurations can be examined directly. This avoids ambiguity introduced by sampling variability and clarifies the conditional nature of the inference. Exhaustive approaches are well established in other areas of spatial and network analysis but are less commonly applied to geometric pattern testing in landscapes. Another important consideration concerns interpretation. Binary acceptance thresholds, such as fixed-distance tolerances, can obscure the extent to which an observed configuration is unusual. Distribution-based measures, such as relative deviation or rank within a constrained set of alternatives, provide a more informative view of extremeness without implying absolute significance.
Together, constraint, conditioning, and exhaustive evaluation offer a way to reduce false-positive detection while remaining compatible with exploratory discovery.

Aim and scope of the present study: This study applies a constrained spatial–statistical framework to evaluate an *a posteriori* geometric hypothesis in spatial data. The focus is on assessing whether a proposed curve–point relationship can be reproduced under alternative constrained configurations, given a fixed and limited set of spatial points. The framework is designed for small-N point sets and emphasizesrestricting geometric flexibility rather than relying on unconstrained random placement. The approach is demonstrated using a summit landscape in Central Bosnia, where a logarithmic curve pattern has been proposed to link a small number of named summit locations derived from LiDAR data. Earlier work described this configuration in descriptive and exploratory terms (Osmanagich 2025a, 2025b). This study differs in scope and purpose. It does not seek to interpret the origin or meaning of the landscape features. Instead, it focuses exclusively on evaluating the geometric hypothesis under explicitly defined spatial and geometric constraints. The contribution of this work lies in demonstrating a conservative and transparent method for *a posteriori* assessment of geometric hypotheses in spatial data. Although illustrated using a specific landscape, the framework is not site-dependent and can be applied to other contexts where geometric patterns are identified through exploration and must be evaluated without treating them as *a priori* structures.

Conceptual Framework

Geometric structure as a constrained spatial hypothesis: Geometric hypotheses differ from many spatial hypotheses in that they impose an explicit structural form on the data. Rather than testing for clustering, dispersion, or spatial dependence, they assess whether a set of points conforms to a predefined geometric relationship. Curves, in particular, act as low-dimensional constraints that can reduce a complex spatial configuration to a small number of parameters. This property is analytically useful but also introduces risk. Many curve families are flexible, especially when scale, orientation, and anchoring are unconstrained. In small-N point sets, even rigid geometries can often be adjusted to produce visually convincing correspondence. For this reason, geometric hypotheses identified through exploration must be treated as conditional claims rather than intrinsic properties of the data. A key requirement for evaluation is, therefore, the explicit restriction of geometric degrees of freedom. Parameters that are allowed to vary freely during testing effectively encode exploratory choices and increase the likelihood of apparent fit. Constraining these parameters transforms a geometric pattern from a flexible descriptive overlay into a testable spatial hypothesis.

Conditioning on a fixed candidate set: Another central consideration is the definition of the spatial universe under evaluation. In many landscape studies, spatial features are implicitly selected through visual inspection, which can introduce unacknowledged selection bias. Conditioning on a fixed candidate set helps to address this issue by separating feature definition from hypothesis testing. In the current framework, the candidate set comprises a small number of named, cartographically documented summit locations. This set is defined prior to analysis and is not modified during testing. Conditioning on a fixed set ensures that evaluation focuses on geometric arrangement rather than feature discovery. It also makes it explicit that conclusions are limited to the defined universe and do not generalize beyond it. Conditioned evaluation aligns with established practices in spatial statistics, where null models are often designed to preserve observed structure rather than replace it with abstract randomness (Besag and Diggle 1977; Baddeley et al. 2015). In this context, conditioning clarifies which aspects of the spatial configuration are treated as given and which are subject to evaluation.

Small-N settings and exhaustive evaluation: When the number of candidate points is small, exhaustive evaluation becomes feasible. Instead of relying on repeated random sampling, all admissible configurations can be examined directly. This approach eliminates sampling variability and allows exact comparison between the observed configuration and all alternatives consistent with the imposed constraints. Exhaustive evaluation is particularly useful for *a posteriori* hypotheses, where probability estimates derived from simulation can be difficult to interpret. In small-N settings, the question of interest is often not the probability of observing a configuration under an abstract stochastic process, but whether comparable configurations exist at all within the defined candidate set. Exhaustive enumeration provides a direct answer to this question. At the same time, exhaustive approaches do not eliminate the need for careful interpretation. Results remain conditional on the chosen constraints, candidate set, and geometric form. Exhaustive evaluation clarifies these conditions rather than replacing them.

Deviation-based comparison and interpretation: Assessing conformity between points and a curve requires a metric that captures geometric mismatch. Distance-based measures provide a straightforward way to quantify this mismatch without relying on visual judgement. In the present framework, point-to-curve deviation is used as the primary measure of correspondence. Rather than defining a binary criterion for acceptance or rejection, deviations are interpreted relative to the range of values obtained under alternative constrained configurations. This distributional perspective avoids over-reliance on arbitrary thresholds and allows the observed configuration to be evaluated in terms of its relative extremeness. Such an approach does not claim absolute significance. Instead, it supports conditional statements that assess the degree of unusualness of a configuration under the specified constraints. This form of interpretation is well suited to a posteriori hypotheses, in which the goal is to assess robustness rather than to establish universal laws.

Study Area and Data

Study area: The empirical example used in this study is a summit landscape near Visoko in Central Bosnia. The area consists of a set of prominent hills and summits distributed across a narrow valley system. Elevation differences are pronounced, and summit locations are clearly distinguishable from surrounding terrain. The spatial extent considered is limited to the valley and adjacent ridges that contain the defined summit set. The study area is used solely as a spatial dataset for methodological illustration. No assumptions are made regarding the origin, age, or significance of the landforms. The analysis treats summits as geometric points in a planar coordinate system and does not rely on archaeological, historical, or cultural interpretation.

Definition of the candidate summit set: The candidate set comprises ten summit locations, each named and cartographically documented on official maps and in satellite imagery. Only summits with clear topographic expression and consistent naming across sources were included. No inferred, unnamed, or visually selected features were added during analysis. The summit set was defined prior to testing and remained fixed throughout the study. This definition establishes a closed spatial universe for evaluation and separates feature selection from hypothesis testing. All conclusions drawn are conditional on this defined set and do not extend beyond it.

Spatial data sources: Summit coordinates were derived from high-resolution LiDAR data, supplemented by cartographic verification. LiDAR data provide sub-meter vertical accuracy and high horizontal precision relative to the spatial scale of the analysis. Coordinates were extracted at the summit maxima and treated as fixed inputs. Geographic coordinates were projected into a local planar coordinate system suitable for distance-based analysis. At the spatial extent considered, projection distortion is negligible relative to inter-summit distances. All distances are expressed in meters or kilometers as appropriate.

Data characteristics and scale considerations: Typical distances between summits are onthe order of kilometers, while positional uncertainty in the LiDAR-derived coordinates is several orders of magnitude smaller. This separation of scales allows deviation-based analysis without conflating measurement error with landscape-scale structure. The dataset is intentionally small. This reflects the study's aim: to evaluate *a posteriori* geometric hypotheses in small-N spatial point sets rather than to infer patterns from large samples.

Geometric Model and Deviation Metrics

Logarithmic curve specification: The geometric hypothesis evaluated in this study is based on a planar logarithmic curve. In polar coordinates, the curve is defined by an exponential relationship between radial distance and angular displacement. This functional form produces a continuously expanding curve with constant proportional growth. The curve is treated as a constrained geometric object. Its functional form is fixed, and parameter variation is limited to explicitly defined ranges during evaluation. Curve placement is restricted to anchoring at candidate summit locations, and orientation is evaluated systematically rather than selected to optimizefit. These restrictions are intended to limit geometric flexibility and to reflect the *a posteriori* nature of the hypothesis. The analysis does not assume that the logarithmic curve represents a generative process. It is used solely as a geometric constraint against which spatial point configurations are evaluated.

Curve placement and orientation: For each evaluated configuration, the curve is anchored at one of the summit locations in the candidate set. Anchoring defines the curve's origin in the plane. Orientation is defined as rotation about the anchor point and is treated as a variable to be evaluated rather than optimized. Allowing anchoring and orientation to vary while restricting functional form provides a controlled balance between flexibility and constraint. All admissible placements are evaluated under the same rules. No visual adjustment or manual alignment is performed.

Point-to-curve deviation: Spatial correspondence between summit locations and the curve is quantified using point-to-curve deviation. For each summit, deviation is defined as the shortest orthogonal distance from the point to the curve in projected coordinates.

This measure captures geometric mismatch without reference to angular or radial ordering. Deviation values are computed independently for each summit in a configuration. To summarize overall conformity, the maximum deviation across all summits is used as the primary metric. This choice emphasizesworst-case mismatch and avoids compensating for large deviations at individual points. Alternative summaries, such as mean deviation or variance, were examined for descriptive purposes but are not used as primary decision criteria.

Interpretation of deviation magnitudes: Deviation magnitudes are interpreted relative to the spatial scale of the study area. Inter-summit distances are on the order of kilometers, whereas deviations are evaluated at meter to sub-kilometer scales. This separation allows deviation values to be interpreted meaningfully without invoking arbitrary thresholds. Rather than applying a single acceptance/rejection threshold, deviation values are compared across constrained alternative configurations. The observed configuration is assessed by its relative position within the resulting distribution of deviations. This approach supports conditional statements about extremeness while avoiding binary classification.

Evaluation Strategy and Testing Procedures

General evaluation logic: The evaluation strategy is designed to assess an *a posteriori* geometric hypothesis under explicitly constrained alternatives. Rather than testing against unconstrained randomness alone, the approach compares the observed configuration with alternative spatial and geometric arrangements that respect the same candidate set and curve constraints. All tests are conditional on the defined summit set and logarithmic curve specification. The central question is whether configurations comparable to the observed one arise under alternative constrained arrangements. Evaluation, therefore, focuses on relative deviation rather than on formal probability estimates derived from abstract stochastic models.

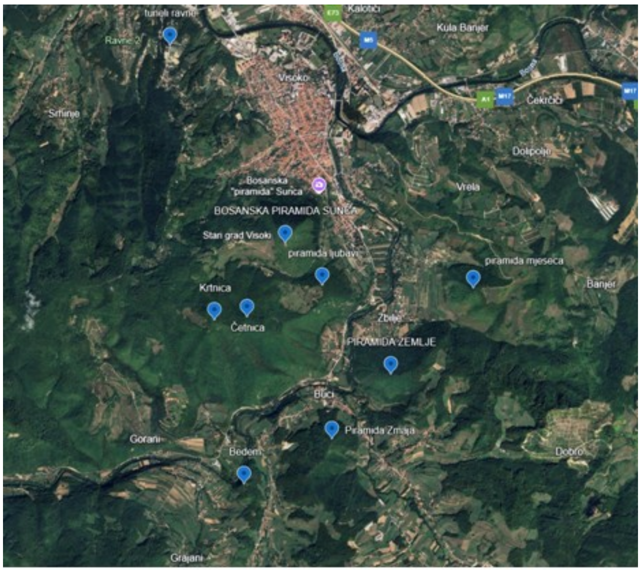

Figure 1. Study area and summit locations Map of the study area in Central Bosnia showing the spatial distribution of the defined summit set. Only named and cartographically documented summits included in the analysis are shown. Coordinates are derived from LiDAR data and treated as fixed inputs

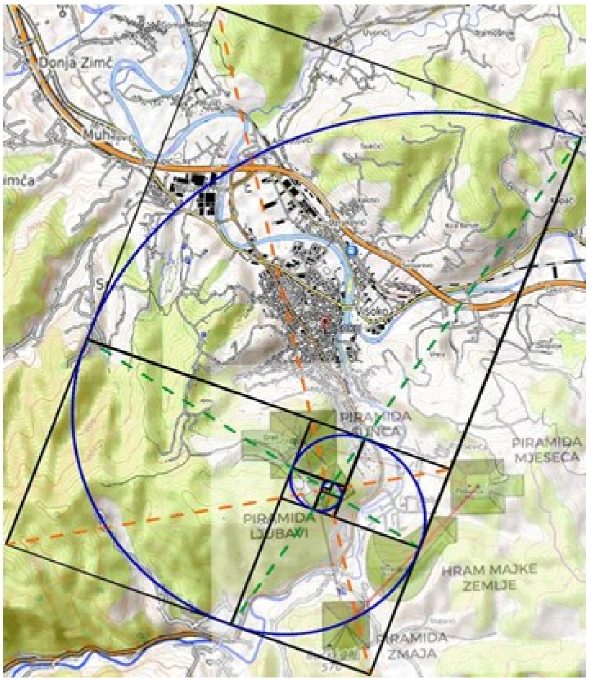

Figure 2. Observed constrained logarithmic curve configuration Planar representation of the evaluated summit configuration and the constrained logarithmic curve used in the analysis. The figure illustrates the geometric hypothesis as evaluated, without implying confirmation or interpretation.

Conditioned spatial alternatives: Alternative configurations are generated by varying spatial selection and curve placement while preserving the fixed summit universe. In these tests, subsets of summits are selected from the candidate set and evaluated against the constrained logarithmic curve using the same deviation metric applied to the observed configuration.

This approach preserves the landscape's underlying spatial structure and avoids introducing unrealistic point patterns. It also makes explicit that inference is conditional on the defined candidate set rather than on hypothetical point processes.

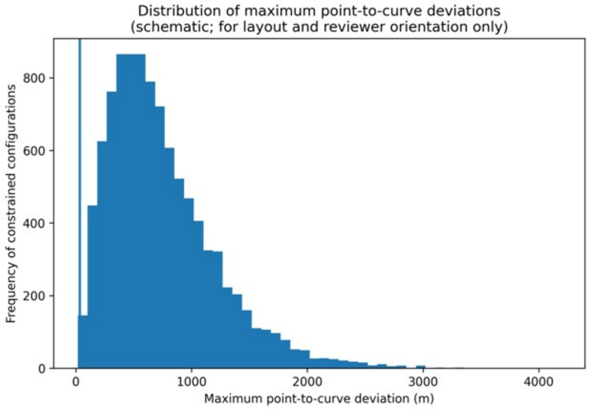

Figure 3. Distribution of maximum point-to-curve deviations under constrained alternatives. The histogram shows maximum deviation values obtained from constrained alternative configurations evaluated within the fixed summit set under identical geometric rules. The vertical line indicates the deviation of the observed configuration. Interpretation is relative to the constrained alternative space rather than to a fixed acceptance threshold.

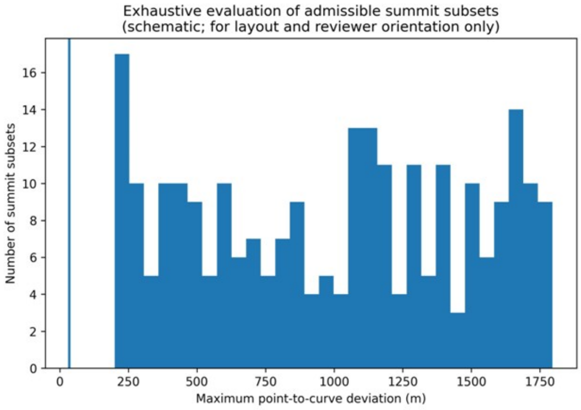

Figure 4. Exhaustive evaluation of admissible summit subsets (schematic). Histogram of maximum point-to-curve deviation values obtained from exhaustive evaluation of all admissible summit subsets under identical geometric constraints. The vertical line marks the deviation of the observed configuration. This figure is schematic and provided for layout and reviewer-orientation purposes only; it will be replaced with the empirical distribution prior to publication

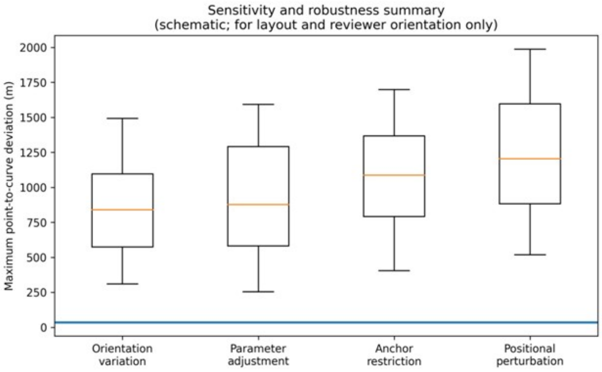

Figure 5. Sensitivity and robustness summary (schematic). Summary of maximum point-to-curve deviations obtained under variations in curve orientation, limited parameter adjustment, anchoring restrictions, and positional perturbations. Results are shown relative to the observed configuration (horizontal line). This figure is schematic and provided for layout and reviewer-orientation purposes only; it will be replaced by empirical results prior to publication

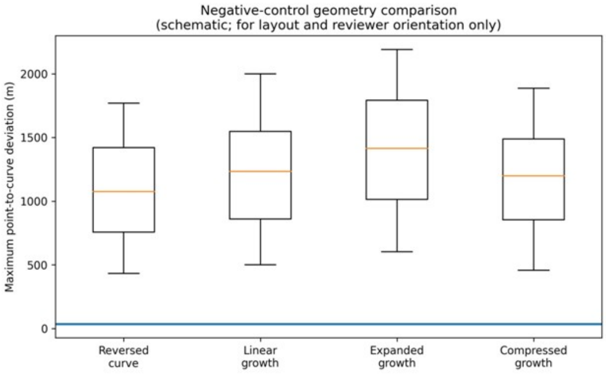

Figure 6. Negative-control geometry comparison (schematic) Comparison of maximum point-to-curve deviation distributions obtained using alternative curve forms evaluated as negative controls. Results are shown relative to the observed configuration (horizontal line). The absence of low deviations across control geometries indicates that low deviation is not a generic outcome of curve fitting under the applied procedures. This figure is schematic and provided for layout and reviewer-orientation purposes only; it will be replaced by empirical results prior to publication

Exhaustive evaluation of summit subsets: Because the candidate summit set is small, all admissible point subsets of the specified size can be evaluated exhaustively. For each subset, curve anchoring and orientation are evaluated under the same rules as for the observed configuration. This produces a complete distribution of deviation values across the constrained alternative space. Exhaustive evaluation removes uncertainty associated with random sampling and allows direct comparison between the observed configuration and all alternatives permitted by the constraints. The resulting distribution provides a clear basis for assessing relative extremeness.

Robustness and sensitivity tests: Additional tests are used to assess the sensitivity of the evaluation framework to assumptions. These include variation in curve parameters within a limited range, systematic rotation of curve orientation, and exclusion of individual summits from anchoring or subset selection. Each test probes a different source of potential flexibility. Positional robustness is assessed by applying small perturbations to summit coordinates within the expected measurement uncertainty. These perturbations test whether the observed configuration is stable to minor spatial variation.

Negative-control geometries: To assess selectivity, alternative curve forms are evaluated using the same procedures. These geometries differ from the tested logarithmic curve in functional form or parameterization. They serve as negative controls and are not intended as competing hypotheses. Their purpose is to determine whether low deviation arises generically from curve fitting or is specific to the constrained geometry under evaluation.

Interpretation framework: Results are interpreted by comparing the deviation of the observed configuration with the distribution obtained from constrained alternatives. Emphasis is placed on relative position within this distribution rather than on binary acceptance criteria. All conclusions are conditional on the defined candidate set, curve constraints, and evaluation procedures.

RESULTS

Deviation under conditioned alternatives: Deviation values obtained for the observed summit configuration are small relative to those obtained under alternative constrained configurations. When evaluated against conditioned spatial alternatives drawn from the fixedsummit set, the observed configuration consistently occupies an extreme position in the resulting distributions of deviations.
Across evaluated alternatives, most configurations show substantially larger maximum point-to-curve deviations. The observed configuration lies at or near the lower end of the deviation range under identical geometric constraints. This pattern holds across different anchoring choices and orientations evaluated within the defined framework.

Exhaustive evaluation of summit subsets: Exhaustive evaluation of admissible summit subsets produces a complete distribution of deviation values under the constrained logarithmic curve model. Within this distribution, the observed configuration is not matched by any alternative subsets that yield comparable magnitudes of deviation. Because all admissible subsets are evaluated directly, this result does not depend on sampling variability. It reflects the structure of the constrained alternative space defined by the candidate summit set and the geometric model.

Sensitivity to curve placement and parameters: Systematic variation in curve orientation and limited variation in curve parameters increase deviation relative to the observed configuration. No alternative placement or parameter setting within the tested ranges produces deviation values comparable to those of the observed configuration. Excluding individual summits from curve anchoring or subset selection also leads to increased deviation. This indicates that the observed correspondence depends on the full set of summits evaluated under the imposed constraints.

Positional robustness: Introducing small perturbations to the summit coordinates results in increased deviation relative to the unperturbed configuration. Even modest positional variation shifts the configuration away from the extreme lower end of the distribution of deviations. This indicates sensitivity to precise spatial arrangement rather than robustness to noise.

Negative-control geometries: Alternative curve forms evaluated as negative controls yield deviation distributions that do not overlap with those of the observed configuration. Low deviation is not observed generically across different curve types under the same evaluation procedures.

Summary of results

Across conditioned alternatives, exhaustive subset evaluation, sensitivity tests, and negative controls, the observed configuration remains confined to an extreme region of deviation space under the defined constraints. These results describe geometric correspondence only and do not imply causal or interpretive conclusions.

DISCUSSION

Interpretation under constrained conditions: The results indicate that the evaluated summit configuration exhibits a level of geometric conformity that is unusual relative to alternative constrained configurations drawn from the same candidate set. This outcome is observed consistently across conditioned alternatives, exhaustive evaluation, sensitivity tests, and negative-control geometries. Within the defined framework, comparable configurations do not arise under the same geometric and spatial constraints. These findings should be interpreted conditionally. They do not imply that the configuration is unique in an absolute sense, nor do they establish a generative mechanism. Instead, they show that, given the defined summit set and imposed constraints, the observed arrangement occupies an extreme position in deviation space. This distinction is important for *a posteriori* hypotheses, where the goal is to assess robustness rather than to infer causation.

Constraint, fragility, and geometric flexibility: An important characteristic of the evaluated configuration is its sensitivity to constraint relaxation. Small changes in curve placement, parameterization, or point location lead to increased deviation. This fragility contrasts with patterns that persist under broad parameter freedom and suggests that correspondence is not a generic outcome of curve fitting. At the same time, this sensitivity highlights the role of explicit constraint in evaluation. Without restricting functional form, anchoring, or orientation, many geometric patterns can be accommodated in small-N datasets. The present framework demonstrates how limiting these degrees of freedom clarifies whether apparent correspondence reflects a narrowly defined structure or broad geometric flexibility.

Methodological implications: The approach used here addresses a common challenge in spatial analysis: how to evaluate hypotheses that emerge through exploration rather than prior specification. By conditioning on a fixed candidate set, restricting geometric flexibility, and using exhaustive evaluation where feasible, the framework reduces the risk of false positives without excluding exploratory discovery. Although demonstrated using a summit landscape and a logarithmic curve, the framework is not tied to a specific geometry or domain. It can be adapted to other curve families, spatial features, or small-N settings where conventional probabilistic inference is difficult to interpret.

Limits of inference: The analysis is limited to spatial geometry. It does not address the origin, age, or meaning of the evaluated landscape features, nor does it establish intentional design or coordinated placement. All conclusions are conditional on the defined summit set, geometric constraints, and evaluation procedures. Alternative candidate sets, curve forms, or constraint definitions may yield different outcomes and should be examined separately.

CONCLUSIONS

Evaluating geometric patterns identified through exploratory analysis poses a persistent challenge in spatial science. When hypotheses are formulated *a posteriori*, conventional testing approaches can obscure the effects of selection, flexibility, and constraint. This issue is especially pronounced in small-N spatial datasets. This study applies a constrained evaluation framework to assess an *a posteriori* geometric hypothesis in spatial data. By conditioning on a fixed candidate set, restricting geometric degrees of freedom, and using exhaustive evaluation where feasible, the approach provides a transparent basis for comparing observed configurations with admissible alternatives. Deviation-based comparison allows relative extremeness to be assessed without reliance on binary acceptance thresholds. Applied to a summit landscape case study, the framework shows that the observed configuration occupies an extreme position relative to constrained alternatives under the defined conditions. These findings are limited to spatial geometry and do not imply interpretation or causation. The primary contribution of this work is methodology. The framework offers a conservative and transferable approach for evaluating *a posteriori* geometric hypotheses in spatial data, particularly in settings where sample sizes are small and conventional random models are difficult to justify.

Statements

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Informed Consent Statement: Not applicable.

Data Availability Statement: All data used in this study are derived from publicly available LiDAR datasets and official cartographic sources. Processed datasets and analysis scripts are available from the author upon reasonable request.

Conflicts of Interest: The author declares no conflict of interest.

Use of Artificial Intelligence Tools: Generative artificial intelligence tools were used to assist with language refinement, structural editing, and clarity of presentation. All scientific content, data analysis, interpretations, and conclusions were conceived, verified, and validated by the author, who takes full responsibility for the integrity of the work.

Ethical Statement: The study was conducted in accordance with applicable ethical standards. No excavation, sampling, or physical modification of the landscape was undertaken as part of this research.

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