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REVIEW ARTICLE

MATHEMATICAL MODELING OF THE EFFECTS OF MECHANICAL VIBRATIONS ON THE HUMAN BODY

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ABSTRACT

Acceptable vibration levels for the various organs of the human body are specified based on the response of an undamped single-degree-of-freedom system subjected to harmonic vibration. The equations that define the vibrating system account for variations in the amplitudes of displacement, velocity, and acceleration as a function of the vibration frequency.

Keywords:
Human Body, Damping, Degrees of Freedom, Vibration.

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INTRODUCTION

The vibrations generated by equipment or machinery commonly used in the workplace have significant effects on the human body. Many pieces of equipment or machinery used to perform work tasks generate vibrations as a direct result of their operation. This includes the operation of large road construction machinery and tractors, as well as, in the case of construction, tools such as jackhammers, whose operating principles inherently involve vibrations and impacts. Although the adverse effects of vibrations on the human body have been known for several decades, this topic has only recently become the subject of study in our country, following its inclusion in legislation. To highlight the need to consider the effects of this phenomenon on the human body, let's say that each organ of the human body can be regarded in and of itself as an independent mechanical system with its own characteristics of elasticity, mass, and damping. This defines different natural frequencies for each organ or group of organs. The "natural frequency" of a mechanical system is the frequency at which it is possible to produce high-amplitude vibrations with little external energy input to the system. Thus, at this "natural frequency"—which is characteristic of the system itself—a mechanical system offers little resistance to movement. It is important to determine acceptable vibration levels for different tissues in order to prevent damage, especially to workers who are occupationally exposed. Figure 1 provides some values for different tissues.

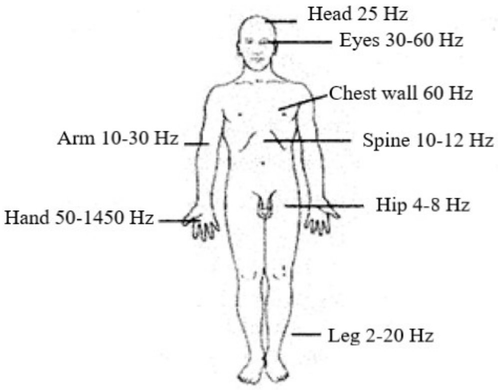

Figure 1. Sensitivity to vibration frequency of different parts of the human body.

Definition of the mathematical model: Acceptable vibration levels are specified based on the response of an undamped system with a single degree of freedom subjected to harmonic vibration. The limits are shown on a graph called a vibration nomogram, which illustrates the variations in displacement, velocity, and acceleration amplitudes as a function of vibration frequency. For harmonic motion, the displacement equation is defined as follows:

$$x(t) = Xsenwt \qquad 1$$

The velocity and acceleration are determined by.

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The velocity and acceleration are determined by.

$$v(t) = \dot{x}(t) = wXcoswt$$

$$a(t) = \ddot{x}(t) = -w^2Xsenwt \qquad 2$$

The relationship between the circular frequency and the linear frequency is stated by.

$$\frac{2\pi}{w} = \frac{1}{f}$$

$$w = 2\pi f$$

substituting w into equation 2

$$\ddot{x}(t) = -(2\pi f)^2Xsenwt$$

Where w is the circular frequency in rad/s, f is the linear frequency in Hz and X is the amplitude of the displacement. The displacement amplitudes, maximum velocity, and maximum acceleration are related to the following expressions.

$$v_{max} = 2\pi fX \qquad 4$$

$$a_{max} = -4\pi^2f^2X = -2\pi fv_{max} \qquad 5$$

Taking the logarithms of equations 4 and 5, we have.

$$ln(vmax) = ln(2\pi f) + ln(X) \qquad 6$$

$$ln(amax) = ln(vmax) - ln(2\pi f) \qquad 7$$

It can be seen that, for a constant value of displacement amplitude X. Equation 6 shows that $ln(v_{max})$ with $(2\pi f)$ and forms a straight line with a slope of +1. Likewise, for a constant value of the acceleration amplitude a_{max}. Equation 7 indicates that ln v_{max} varies with $ln(2\pi f)$ as a straight line with a slope of −1. These variations are shown as a nomogram in Figure 2. Therefore, each point on the nomogram represents a specific harmonic sinusoidal variation. Since the variation imparted to a human or a machine consists of many frequencies, the values of the square root of the mean of the squares of $x(t)$, $v(t)$ and $a(t)$ are used in specifying vibration levels.

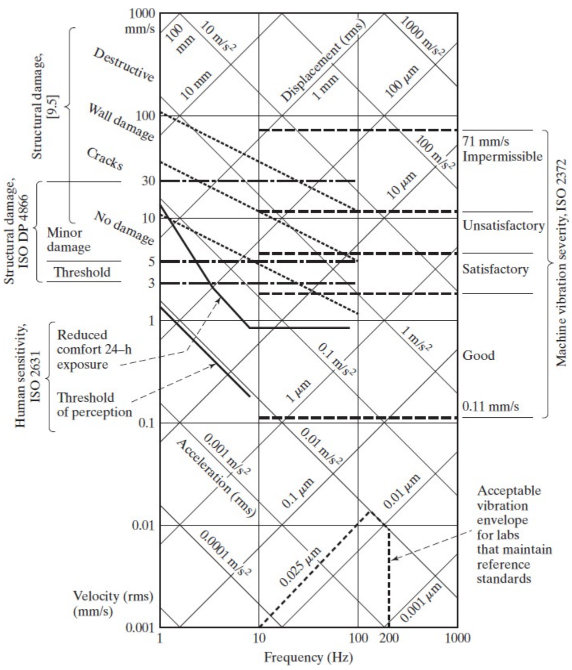

Figure 2. Vibration criteria nomograph

Using Equations 1 through 7, as previously defined, it is possible to determine the basic vibration characteristics for different parts of the human body. To illustrate the application of the expressions in this treatise, we can consider a specific example: Let us consider the angular motion of the forearm, which supports a mass $m0$. During the motion, the forearm rotates about the joint at pivot 0 under muscle forces modeled as a force generated by the triceps $(c_1\dot{x})$ and a force generated by the biceps $(c_2\emptyset)$, where c_1 and c_2 are constants and $\dot{x}$ is the velocity at which the triceps lengthens or contracts. We can represent the arm as a uniform rod of mass m and length l. Using this information, we will derive the equation of motion for the forearm for small angular displacements $\emptyset$ and the natural frequency w of the forearm.

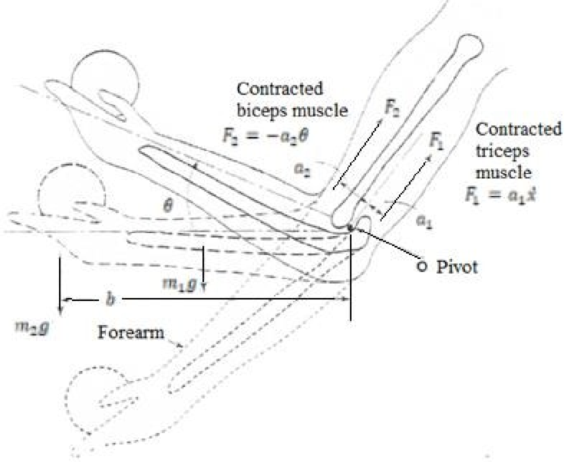

Figure 3. Angular movement of the forearm with a mass on the wrist

The kinetic energy of the system is.

$$T = T_r + T_b = \frac{1}{2}(ml^2/3)\dot{\emptyset}^2 + \frac{ml^2\dot{\emptyset}^2}{2}$$

The potential energy of the system is.

$$U = U_r + U_b = \frac{mgl(1-cos\emptyset)}{2} + \frac{mgl(1-cos\emptyset)}{2}$$

From the equation of motion, we have.

$$\frac{d}{dt}(T+U) = 0$$

Then

$$(m + m/3)l^2\ddot{\emptyset} + (m + m/2)glsen\emptyset$$

For small angles, the equation of motion is defined as.

$$\ddot{\emptyset} + \left[\frac{m + m/2}{m + m/3}\right]\left(\frac{g}{l}\right)\emptyset = 0$$

Finally, the natural frequency of the system is.

$$w_n = \sqrt{\frac{(m + m/2)m}{(m + m/3)l}}$$

The foregoing demonstrates that mechanical vibration equations can be applied to define the various vibration systems of the different organs of the human body under different occupational conditions.

CONCLUSION

The behavior of the various organs of the human body when exposed to vibrating mechanical systems can be analyzed using vibration analysis theory. The degree of complexity of the analytical models will depend on the elements to be considered in the specific organ. When the mathematical models are too complex to solve, finite element software can be used to solve the equations. The scientific study of the effects of mechanical vibrations on human tissue makes it possible to eliminate or minimize the potential harm that vibrations could cause to the health of people exposed to them in the workplace. Exposure to uncontrolled vibrations, without understanding their effects on different tissues of the human body, can be very dangerous, as damage to vital organs will radically alter the lifestyle of those suffering from a condition caused by this exposure and, in some situations, may put people's lives at risk, depending on the degree of exposure.

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